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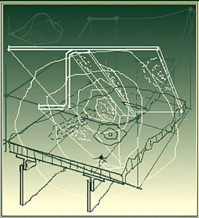

Composite Structures, Design, Safety and Innovation

B.F. Backman

COMPOSITE STRUCTURES, DESIGN, SAFETY AND INNOVATION

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Preface

This book is intended for Structural Safety Professionals in industry, government and academia and for advanced graduate students with an ambition to better understand the challenges of the design process for composite structures.

The writing of this book is influenced by the continuing, innovative aspects of composite structure. A steady stream of new materials, processes and structural concepts has made established empirical structural design approaches obsolete and the absence of pertinent service experience has forced a rethinking of the role of safety in structural design. Traditional structural design is based on implicit structural considerations like "allowables," safety factors and margins of safety. Innovation is the state of the evolution of composite structures and explicit safety measures have to be introduced, like probability of an unsafe state, or innovation will not be manageable from a safety standpoint. The states of uncertainty caused by the "new" has to be dealt with in terms of risk management, monitoring of safety levels and control processes for "course corrections" in service.

A well-defined system of safety responsibilities that are in agreement with "future" regulations for composite design, manufacture, maintenance and operation must be based on current situations in the service environment that not only is location dependent but also change in time. Requirements must be kept current and well defined, while means of compliance must be adaptable.

The future in structures belongs to a large degree to composites, but only if introduced through safe innovation and explicit safety measures.

This book sets the stage for the continued dialog. A number of examples that use required vehicle safety to discuss consequential orders of magnitudes to describe the realism in applying random considerations to practical design challenges in an arena that has been ferociously deterministic. These examples touch on the bounds of what is possible in a rational approach to satisfying explicit safety requirements and can be used as a basis for homework, if used in class. Parametric variations of what is needed, what is required and what is possible are effective approaches to understanding the practical aspects of engineering design of safe composite structure. Chapters 2 and 3 contain detailed studies of what may be considered realistic details of accidental damage scenarios, and can be part of the accidental damage design criteria foundation, but may be best revisited on a case-by-case basis in practical design work.

The author especially expresses his gratitude to Dr James H. Starnes formerly of NASA Langley for his unwavering support of this work.

CONTENTS

Chapter 1

Introduction

Safe composite structure is an important part of the development of modern flight vehicles. It involves many disciplines, like Material science, Structural engineering, Manufacturing technology, Maintenance engineering, Inspection technology and Operation on the ground and in the air. Structural safety is a necessary requirement in the achievement of successful innovation and efficient design development.

The event "Safe composite structure," S, is a joint event made up of at least these four sub-events:

$$S = D\ I\ M\ O$$

where

- D is the event "Safe structural design";
- I is the event "Safe maintenance," which includes proper inspection and repair methods;
- M is the event "Safe manufacturing," which includes performing according to specifications, drawings and instructions;
- O is the event "Safe operation," which includes abiding by operating procedures and flight manuals; e.g. not exceeding limit external loads.

The probability of safe structure can be expressed as:

$$P(S) = P(D|I\ M\ O)P(I|M\ O)P(M|O)P(O) \quad (1.1)$$

according to the multiplication rule in the probability theory.

- The first factor, $P(D|I\ M\ O)$ is the probability of a safe design, given safe maintenance, safe manufacturing and safe operation;
- The second factor, $P(I|M\ O)$ is the probability of safe maintenance, given safe manufacturing and safe operation;
- The third factor, $P(M|O)$ is the probability of safe manufacturing, given safe operation; and
- The fourth factor, $P(O)$ is the probability of safe operation.

This book will primarily focus on the first factor, referring to "safe composite structural design." Structural integrity will be the design goal and the safety measures will be based on the development of design criteria with explicit safety constraints.

1.1. TRADITIONAL DESIGN IN AEROSPACE

The history of structural design during the last sixty years has been the history of "Riveted-Aluminum-Skin–Stringer-Constructions." And while extraordinary progress has been made in materials and processes, design has remained mainly an empirical quest, finding its incentive to improvements in service experience with changes in "rules of thumb" and with very cautious introduction of change. The approach has become very rooted in what "has been," and design methods have become of limited value in the pursuit of innovation. The epoch has produced very safe vehicles, but the lessons learned are not applicable to a "composite world" except as a reminder of the necessity of avoiding the unexpected.

The design focus was for a long time on ultimate strength (static strength with a factor of safety of 1.5), with a complement of fail-safety criteria (limit load capability for one failed load path). The introduction of fatigue design evolved as a part of reactions to service experience. Damage tolerance (based on fracture mechanics) also evolved through service experience, but became mainly a way to design inspection programs. This role of damage tolerance was a result of material and fastener improvements that achieved capabilities producing residual strength levels that matched a two-thirds ultimate capability for traditional damage types and sizes.

So the "aluminum era" produced structures with good static strength, a steadily improving fatigue performance, fail-safe detail designs and components and damage tolerant performance.

The rules of thumb for design that emerged had "metal flavor." The empirical design methods are specific to aluminum. The test methods for allowable values and design data also are specific to metals. The present situation is unique to aluminum (especially for commercial, large airplanes). It can be adapted to other metals and there are several successes to point at; e.g. titanium. However, the bulk of this knowledge is not directly transferable to non-metallic structures.

1.2. CONVENTIONAL SAFETY IN AEROSPACE

Safety in conventional structure is not measured in explicit terms because of the evolutionary nature of the field. Instead it has been implicitly achieved through establishing and maintaining Structural Integrity. A review of the regulations and practices reveals that the following types of integrity are part of the practices in the present day "metal world":

- Ultimate strength integrity;
- Fail-safe integrity;
- Damage tolerance integrity;
- Discrete source integrity.

The compliance process is designed to demonstrate that the intended capabilities are achieved, and that is considered as "proof that adequate safety has been achieved."

So, integrity has become the foundation of today's safety in the aluminum world, and this approach has credibility for designs, with substantial service records for the type of structure in question.

1.3. TRENDS IN INNOVATION OF AEROSPACE STRUCTURES

Development of military and space applications is on a steady course toward ever-improving performance. However, commercial vehicles development (especially in the "Large airplane category") is struggling with costs associated with very marginal improvements.

In this arena, composites represent a very powerful potential for substantial advances. Next generation vehicles require reductions of weight, drag and costs to succeed.

Composite structures could be large contributors to such advances; in weight reduction through high strength-to-weight ratios, in drag reduction through their adaptability to "sleeker geometries," and in cost reduction through new processes and advanced structural concepts (beyond skin–stringer construction).

At the same time, the demands for safer airplanes, both domestically and internationally, have been raised by the general public, government agencies and the safety conscious engineering professionals. So safety improvements are high up on the agenda in technology development organizations and, of course, in political circles. These trends, which are very healthy for technology, point to "Better safety and less adverse service experiences."

1.4. COMPOSITES

Structural polymeric composite materials are members of a very inhomogeneous family; from the first generation brittle Epoxies and T300 Carbon fibers to modern toughened Epoxies with much improved fibers, including Polyimides and a multitude of "advanced" fibers and resin types from thermosets to thermoplastics. Each member often exhibits different responses and failure mechanisms.

This diversity contributes to make true accumulation of "Service experience" difficult, and often not transferable to "the next material system." Consequently, composite structures have to be designed "without existing service experience."

The lack of verified structural design methods for new materials, the missing "lessons learned" in the "Safety arena," the non-transferable empirical know-how in testing and design data and the often occurring surprises inherent to new materials, all have made it necessary to look for an additional means (other than service experience) to assure safe structure.

Often, new vehicles come with new materials, new processes, and/or new structural concepts and all of these contribute toward making composite design dependent on the development of new methods in design, testing and validation.

One way to fill in the hole in safety, that lack of service experience may leave, is to develop "explicit safety constraints" based on, "Safety measures" developed from engineering principles and insights.

Despite the diversity, there are common threads in the behavior of composite materials. One very consistent characteristic is their changes in the responses and failure mechanisms due to damage, which makes damage tolerance (e.g. residual strength) a critical feature. Figure 1.1 shows a typical "allowables-interpretation" of residual strength.

The surface represents a specific "Probability Level," $\Pr(RS < RS_A) = p$, where RS_A represents the surface in Figure 1.1. A typical value of p among the allowables

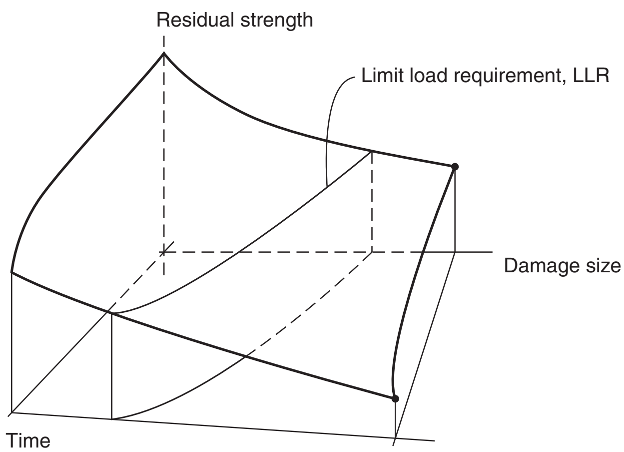

Figure 1.1. Residual strength.

is 0.10 (*B*-value). A large part of the design focus on safety of composite structure involves producing ‘‘quality information’’ for Residual Strength of Damaged Structure, and Figure 1.1 represents data central to structural safety, especially for ‘‘Damage Tolerance Critical Structure.’’

The requirement for composite structures to retain structural integrity in the presence of damage is a central safety feature. So the type of data, residual strength, shown in Figure 1.1 becomes of utmost importance, especially as even ‘‘ultimate integrity’’ involves damage.

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Chapter 2
Structural Design

Design of composite structures has damage tolerance as its major challenge, which means that design criteria are location dependent. Modern designs should address the challenge separately for each Principal Structural Element, PSE. The concept of PSE is accepted in aerospace practices and regulations and serves well as a basis for developing different damage scenarios. The term PSE defines a principal structural segment, the failure of which would result in loss of the vehicle. It turns out that the pursuit of damage tolerant designs will involve residual strength, damage growth rates, damage resistance and fail-safety. It will also be shown how damage tolerance becomes the cornerstone of Structural Safety.

Considering that both established practices and regulations require structural integrity with manufacturing flaws, accidental damage and effects of discrete source events (e.g. bird-strikes) present, it is not surprising that damage tolerance becomes the last bulwark of safety. A safe structural design of a PSE must be based on a realistic assessment of practical damage scenarios. Scenarios that involve definition of threat, initial damage, detectability, damage growth and type of inspection must be part of both design process and design criteria. This chapter contains a cavalcade of possibilities and a display of typical and necessary orders of magnitudes.

Damage tolerance (except for the discrete source events) integrity requires that a limit load capability be maintained during the life of the vehicle. Limit load is defined as "the largest load expected in service," and it turns out for composites that damage tolerance requirements are often more severe than ultimate strength requirements. So both structural design and structural safety are very dependent on damage tolerance.

2.1. DAMAGE TOLERANCE

Damage in service is mostly the result of random events. Location of impact, shape of the impacting object, its size, its inertia, speed and direction are all random variables. The size and severity of damage are random variables. Residual strength is a function of damage size, severity and time. Figure 2.1 shows an "allowables-like" representation (with specific probability level). A probability density function is shown for a point in time and a specific damage size. This figure defines residual strength as a function of time. Damage growth and property degradation, due to environmental effects, depend on time. Consequently, safety level is a function of time.

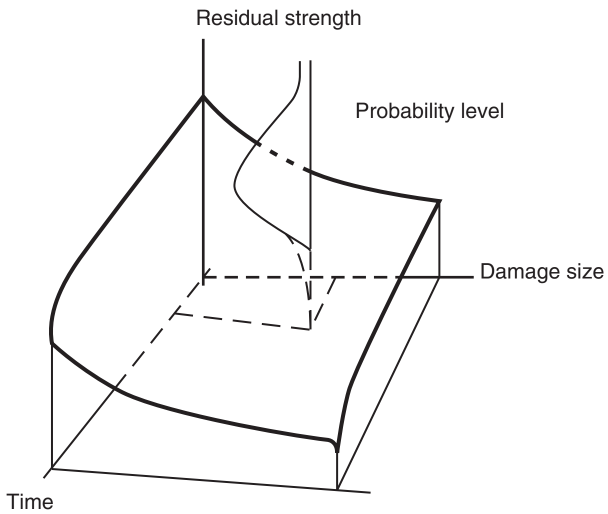

Figure 2.1. Residual strength.

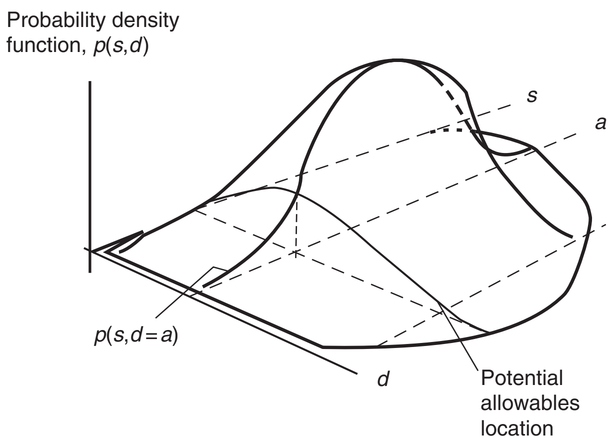

Figure 2.2. Strength, s and damage size, d.

The design of a PSE requires either allowable values on predetermined probability levels or probability distributions (e.g. scaled up from coupon or element data). Figure 2.2 shows an example of a probability density function for damage size and residual strength.

The time effects can be introduced parametrically (in the probability density function once characterized). When the integrity definition contains damage range and strength limits, a quite manageable requirement can be imposed. The implied allowable value definition in Figure 2.2 could be made flexible based on what the specific PSE requires, and it could also be made global as the definition applies to material allowables.

The most effective way, in many cases, is to make it dependent on lay-up and *t*-bar (total area of a specific concept) and standard damage criteria, which could make it a tool for sizing the structure.

2.2. STRUCTURAL INTEGRITY

Each PSE has its own set of integrity requirements based on the criticality situation at each location. However, design criteria can have a common set of goals. Two types are part of every structural design process. One set is based on the nature of the loads and deals with three types:

Ultimate load integrity; static strength;
Limit load integrity; damage tolerance;
"Get-home" load integrity; discrete source damage resistance and damage tolerance.

Ultimate load integrity is the foundation of the classical structural design process, which employs a 1.5 factor of safety to design loads. Composite structure, however, is often more critical for limit loads with damage present. Figure 2.3 illustrates the nature of composite structure criticality.

The described situation (damage tolerance criticality) is the typical case for a composite structure, due to requirements of tolerance to accidental damage. The

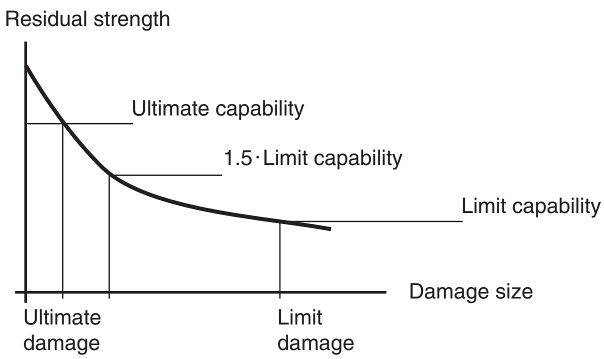

Figure 2.3. Damage tolerance criticality.

ultimate applied stress, for this case is,

$$f_{\text{ult}} = 1.5\, f_{\text{lim}} = \frac{F_B}{1 + \text{MS}} \tag{2.1}$$

where F_B is the B-value ultimate allowable.

So, the ultimate requirements are not dominating in many rational composites designs. However, in combinations of composites and metals and in some applications of composites, it remains important.

It is also important in Fail-safe detail design. For the case of a "lost load path" the "remaining" structure must be able to sustain at least ultimate internal loads (limit external loads).

Limit load integrity deals with structural requirements in the presence of damage, especially damage that is not immediately detected. Damage tolerance in the structure of composites is a major structural design "driver."

Finally, "get-home load" integrity applies to discrete source damage, which is a common term for damage inflicted by especially identified events, like e.g. bird-strike or turbine-blade impact. The event is assumed to be violent enough to alert the pilot, and a reduced load level (often 70 per cent of limit load) is used for the design. Damage resistance is an important aspect of this type of integrity.

The second set of integrities is:

Damage tolerance integrity; limit load capability;
Fail-safe integrity; ultimate (or more) internal load capability;
"Discrete source" integrity; get-home load capability.

This set is directly tied to the required design features and recognizes that:

"The largest load expected in service" is limit load.

These integrities are the basis for structural design and the foundation for structural safety. It will be shown that they form a natural set of requirements for explicit safety-based design constraints. Achieving and sustaining structural integrity is the major objective of the design of safe structure. An acceptable level of integrity, U_{iT}, at time T in location i, involves the following sub-events:

X_{iT}: Damage is not present in location i at time T;
D_{iT}: Damage size, D_s, is less than maximum allowed damage, "MAD";
B_{iT}: Residual strength, RS > limit load requirement, "LLR" at T and i.

Acceptable integrity, U_t at t with location i implied is,

$$U_t = X_t \cup \left(\overline{X}_t U_{\mathrm{Dt}} U_{\mathrm{Ft}} U_{\mathrm{Gt}}\right) \tag{2.2}$$

where

$\overline{X}_t$ is the complement to X_t;
U_{Dt} is the acceptable damage tolerance integrity;
U_{Ft} is the acceptable fail-safe integrity;
U_{Gt} is the acceptable "get-home" integrity.

The probability of an acceptable integrity at t:

$$P(U_t) = P(X) + P\left(U_{\mathrm{Gt}}|\overline{X}_t U_{\mathrm{Ft}} U_{\mathrm{Dt}}\right) P\left(U_{\mathrm{Ft}}|U_{\mathrm{Dt}}\overline{X}_t\right) P\left(U_{\mathrm{Dt}}|\overline{X}_t\right) P\left(\overline{X}_t\right) \tag{2.3}$$

which for unacceptable integrity, can be written as,

$$P\left(\overline{U}_t\right) = P\left(\overline{U}_{\mathrm{Gt}}|\overline{X}_t U_{\mathrm{Ft}} U_{\mathrm{Dt}}\right) + P\left(\overline{U}_{\mathrm{Ft}}|\overline{X}_t U_{\mathrm{Dt}}\right) + P\left(\overline{U}_{\mathrm{Dt}}|\overline{X}_t\right) \tag{2.4}$$

Assuming that the factors in Eq. (2.3) are of the order of magnitude 10^{-3}, or less, this is a good approximation.

2.2.1. Damage tolerance integrity

Damage tolerance integrity assures "Limit Load Capability," LLC, up to a maximum damage size, MAD, larger than what is considered "Easily Detectable Damage," EDD, by pertinent inspection method. Figure 2.4 illustrates a typical case. The figure can be interpreted as describing n individual load paths. The damaged

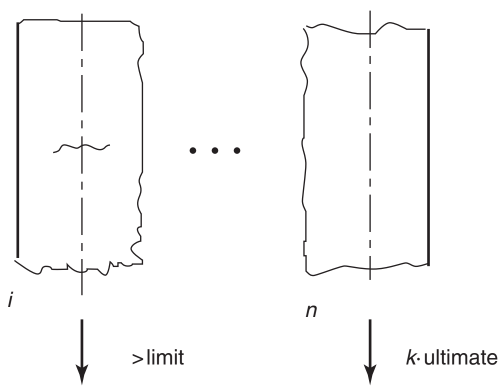

Figure 2.4. Damage tolerance integrity.

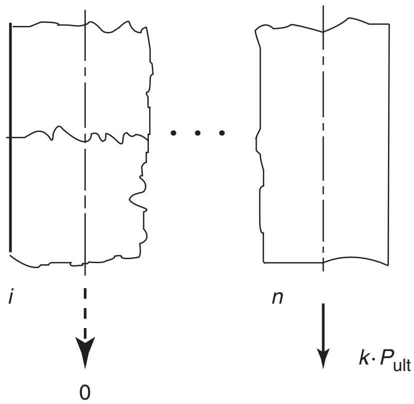

Figure 2.5. Fail-safe integrity.

load path has a preserved LLC and the remaining load paths have minimally an "Ultimate Internal Load Capability." For the common case of "dual load paths," $k = 1.33$.

2.2.2. *Fail-safe integrity*

The objective of fail-safe integrity is to assure LLC with one load path severed. Figure 2.5 illustrates a typical situation considered in a fail-safe design. If the structure is designed fail-safe (a necessary requirement for *B*-value allowables) we find that the design load, P_{crit} is:

$$n = 2: \quad k = 1.33$$
$$n > 2 :\rightarrow 1 \leq k \leq 1.33$$

If we can conclude the load capacity of load path i, P_i , is $P_i > 0$, then the following is true:

$$P\left(\overline{U}_{\text{Ft}} | \overline{X}_t U_{\text{Dt}}\right) = 0$$

If damage tolerance integrity is given in the presence of damage in a specific load path, the probability of loss of fail-safe integrity is zero, if not, additional damage could be included.

2.2.3. *"Get-home integrity"*

"Get-home" integrity can be assured either with adequate residual strength in the damage load path or with redistribution to the alternate paths. Figure 2.6 shows

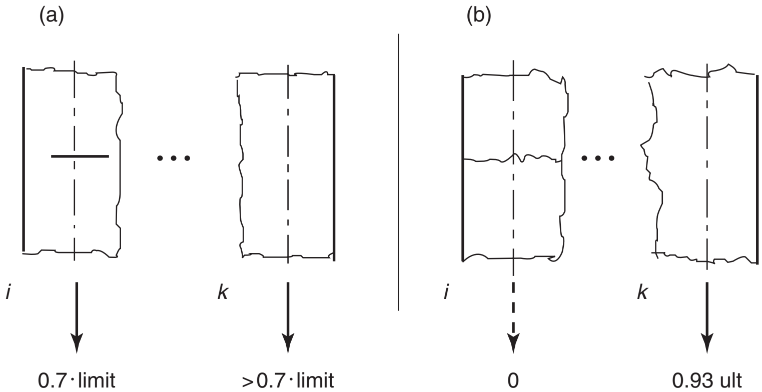

Figure 2.6. Two types of "get-home" integrity.

the two types of "get-home" integrity. Alternative "b" – worst case – with only one alternative load path. If integrity is established for both cases, then as shown in Figure 2.5,

$$P\left(\overline{U}_{\mathrm{Gt}}|\overline{X}_t U_{\mathrm{Dt}} U_{\mathrm{Ft}}\right) = 0 \tag{2.5}$$

the dominance of damage tolerance integrity is validated.

2.3. EXPLICIT DESIGN CONSTRAINTS

In order to establish design constraints, we have to return to the definition of an "Unsafe State" of a PSE. The basic definition is described in Figure 2.7.

2.3.1. Damage tolerance constraint

Figure 2.7 shows a "detected" branch, D, and a "non-detected," branch ND, and the "integrity" axis shows the level of integrity in these two zones, one acceptable and the other unacceptable. The "Unsafe," shaded area, represents an "Unacceptable level of integrity that is undetected." The design objective is to keep the probability of being in the "unsafe zone" small, which could be expressed as:

$$p = P\left(\overline{S}_T\right) = P\left(\overline{U}_T \overline{H}_T\right) = P\left(\overline{H}_\tau \overline{H}_T \overline{U}_T\right) \leq p_r \tag{2.6}$$

Here, time T represents a major inspection and time τ the previous major inspection. The probability of damage undetected in two consecutive major inspections with unacceptable level of integrity at the second, p can be translated to structural design criteria once p_r is set.

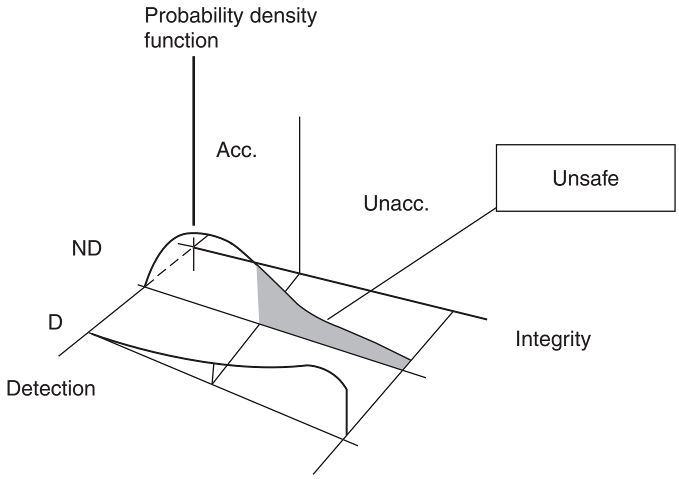

Figure 2.7. Unsafe state.

The previous section demonstrated the importance of damage tolerance integrity, which leads to a "tie-in" with structural properties, and can be expressed as:

$$P(\overline{U}_T) = P(\overline{U}_{\mathrm{Dt}}|\overline{X}_T) = P(\overline{B}_T|D_T\overline{X}_T) + P(\overline{D}_T|\overline{X}_T) \tag{2.7}$$

where the sub-events are:

B_T: Residual strength, RS is larger than Limit Load Requirement, LLR; RS > LLR;
D_T: Damage size, D_s is smaller than Maximum Allowed Damage, MAD; $D_s <$ MAD;
$\overline{X}_T$: Damage is present;

and in general $\overline{Y}$ is the complement to Y.

The first term on the right-hand side of Eq. (2.7) represents the probability that the residual strength is less than the limit load requirement, given that the damage size is less than the maximum "allowed" damage. The second term represents the probability that the damage size is excessive.

The first term therefore presents both a probability requirement and a maximum stress requirement; damage tolerance requirement. The second term is a damage resistance requirement. Both can be used as design criteria.

The use of this definition of "unsafe" begs the question of why. We will look at the probability of surviving from one major inspection to the next for a range of inspection periods of 1000 to 3000 flights. Table 2.1 illustrates survival probabilities.

Table 2.1. Probability of survival for *n* flights after loss of integrity

1 random	500	1000	2000	3000
0.9	0	0	0	0
0.95	0	0	0	0
0.99	0.0063	0.00004	0	0
0.999	0.6083	0.37	0.135	0.05
0.9999	0.95	0.90	0.82	0.74

It is seen in the table that loss of integrity should be avoided, and the probability of an unsafe state, as defined here, should be kept very low.

2.3.2. Damage growth rate constraints

The above definition gives us the lower level of probability that is brought on by major inspections. How the probability grows between inspections depends on the type of PSE, and will be described later. However, Example 2.1 will illustrate one of the possibilities.

Example 2.1: Suppose that the PSE in question is not accessible during preflight inspections and, in this case, not exposed to accidental damage in service. Assume PSE is designed so that damage growth is slow (e.g. grows from region 4 to 5 in two inspection intervals (see Figure 2.9)). Figure 2.8 describes the timeline.

We will look at the probability of an "Unsafe State" during flight k,

$$P(\overline{S}_k) = P(\overline{X}_T\overline{U}_T\overline{H}_T\overline{U}_k\overline{H}_{1k}K_{1k}) + P(\overline{X}_T U_T\overline{H}_T D_{4T} D_{5k}\overline{U}_k\overline{H}_{1k})$$
$$+ \sum P_k(X_T U_T Y_i D_{4i} D_{5k}\overline{H}_{1k}\overline{U}_k)$$

where the following events are involved: ($\overline{X}$ is the complement to X)

X_T: Damage is not present at T;
U_T: The structural integrity is acceptable at T;

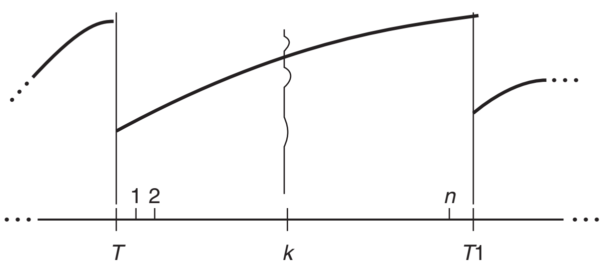

Figure 2.8. Probability of an unsafe state between inspections.

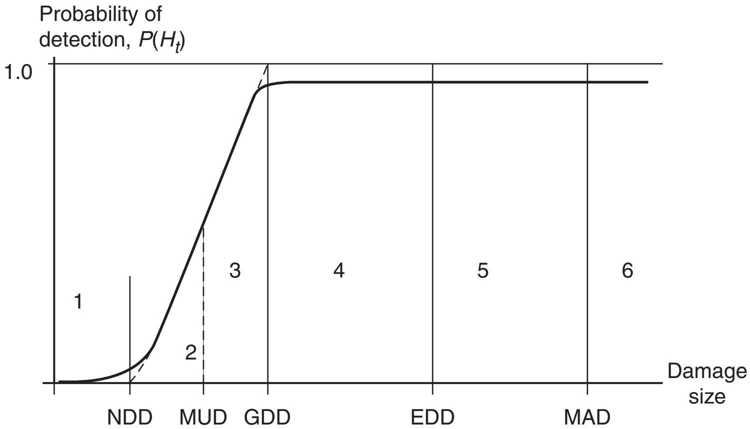

Figure 2.9. Damage size regions 1–6.

H_T: Damage is detected at T;
H_{1k}: Damage is detected between flight 1 and k;
K_{1k}: The PSE survives flight 1 through k;
D_{4T}: Damage size is GDD $< D_s <$ EDD at T (see Figure 2.9);
D_{5T}: Damage size is EDD $< D_s <$ MAD at T;
Y_i: Impact at flight i.

The first term on the right-hand side is very small, if the "walk-around" inspections are efficient and k is larger than thousand flights or survival is very low for a whole period with lost integrity. The term can be expanded as,

$$P\left(\overline{U}_k|\overline{H}_{1k}\overline{X}_T\overline{U}_T\overline{H}_T K_{1k}\right)P\left(\overline{H}_{1k}|\overline{X}_T\overline{U}_T\overline{H}_T K_{1k}\right)P\left(K_{1k}|\overline{X}_T\overline{U}_T\overline{H}_T\right)P\left(\overline{X}_T\overline{U}_T\overline{H}_T\right)$$

The first factor is equal to one. The second depends on the quality of the preflight inspections. The third one deals with survival after lost integrity. Finally, the last one represents the lower bound in the risk management, LB.
The second term deals with growth and detection during "preflight" inspections. The third term deals with impact events in service and their detection. Details will be discussed in the next section.

2.3.3. *Damage resistance constraint*

The third term (the sum) represents accidental damage in service, and one term of the sum can be expanded as an array of factors which includes,

$$P\left(D_{5k}|\overline{Y}_i D_{4i}\right) \quad \text{and} \quad P\left(D_{4i}|\overline{Y}_i\right)$$

where the first factor is controlled by "growth rates" and the second one by damage resistance.

So, the primary design constraints introduced into the design process apply requirements to:

Minimum residual strength (the sizing, the material and the lay-up choices);
Minimum damage resistance (the detail design and sizing);
Maximum growth rates (the detail design and sizing).

Figure 2.8 shows a foundation of what could become a Risk Management process that would control the maximum value by adjusting the inspection intervals and methods, as new data and information become available during service.
The "Explicit Safety-based Design Constraints" can be expressed as:

Residual strength: $\Pr(\mathrm{RS} < \mathrm{LLR}|D_5) < p_r$;
Growth rate: $\Pr(\mathrm{GR} < r) < p_g$, where $r = L/2n$, where $L = \mathrm{MAD} - \mathrm{EDD}$;
Damage resistance: $\Pr(D_{sa} < \mathrm{EDD}) < p_a$, where D_{sa} is initial damage size due to accidental damage.

The right-hand sides in these inequalities are derived from the airplane safety objectives, which for example could be "only one unsafe flight in 100 000." Example 2.2 illustrates a typical case.

Example 2.2: We identify the special influences on the probability of an unsafe flight:

Influence	Probability	
	Effect	Total
Airplane consideration: One unsafe flight in 100 000	10^{-5}	
Structural share, 10%	10^{-1}	10^{-6}
Share of structural design, 0.25	0.25	$0.25 \cdot 10^{-6}$
Assume 50 PSEs in the airplane, 0.02	$2 \cdot 10^{-2}$	$0.5 \cdot 10^{-8}$

The probability of an unsafe flight due to structural problems was shown in Eq. (2.6) to be, after expansion:

$$P(\overline{H}_\tau)P(\overline{H}_T|\overline{U}_T)P(\overline{U}_T) \leq p_r \tag{2.8}$$

A definition of damage tolerance rating, DTR, in design of structure is contained in this equation:

$$P(H) = \frac{1}{1 + 1^{-\mathrm{DTR}}}$$

and a very common minimum requirement for PSEs is,

$$5 < \mathrm{DTR}_{\min} < 6$$

which yields $P(\overline{H}_T) = 10^{-2}$.

The second factor, which deals with not detecting damage large enough to compromise structural integrity, would often turn out to be of the order of magnitude,

$$P(\overline{H}_T|\overline{U}_T) \approx 10^{-3}$$

So the resulting constraint is:

$$P(\overline{U}_T) \leq 0.5 \cdot 10^{-3}$$

An interesting comparison with undamaged structure designed for B-value allowables, F_B for normally distributed strength is the following "limit consequence"

$$10^{-5} \leq \Pr\left(s \leq \frac{F_B}{1.5}\right) \leq 10^{-4}; \text{ for } 0.05 \leq C_{\mathrm{v}} \leq 0.10$$

where C_{v} is the coefficient of variation. So, the order of magnitude seems to be realistic.

2.3.4. Damage scenarios for Principal Structural Elements, PSE

Table 2.2 shows an assortment of damage inflictions, growths and detections that could be considered the basis for individual design criteria for PSEs.

The first row, e.g. describes a situation where walk-around inspections are possible, accidental damage during service is quite possible, accidental damage during maintenance can happen, undetected accidental damage during production is a possibility, and slow damage growth has been achieved through detail design. We will start investigating the relation between unsafe state and type of PSE.

Type 3 represents a relatively simple situation and the first example will focus on that case.

Table 2.2. PSE characteristics and damage types

Scenario	Walk-around	Acc. in service	Acc. in maintenance	No growth	Slow growth	Acc. in production	Degradation
1	•	•	•	–	•	•	–
2	•	•	•	•	•	•	–
3	–	–	•	–	•	•	–
4	–	–	•	•	–	–	–
5	•	•	•	–	•	•	•
6	•	•	•	•	–	•	•
7	–	–	•	–	•	•	•
8	–	–	•	•	–	•	•

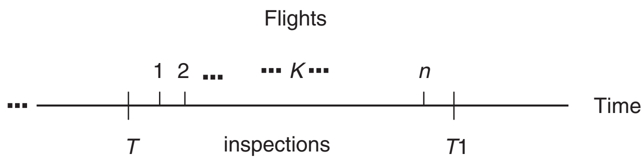

Figure 2.10. Time definition for flights and inspections.

Example 2.3: This case deals with two types of accidental damage; in maintenance and in production. Two events can be identified. The timing of inspections and flights is shown in Figure 2.10. There are n flights in an inspection period (T,T_1).

The following events will address the safety threat at flight k due to accidental damage during maintenance, $X_T U_T Y_{T1} D_{41} D_{5k} \overline{H}_{1k} \overline{U}_k$. It can be defined as,

X_T: No damage was present at time T;
U_T: The state of integrity was acceptable at T;
Y_{T1}: An accidental damage was inflicted during the period T to 1;
D_{41}: The damage size was in region 4 at flight 1;
D_{5k}: The damage size was in region 5 at flight k;
$\overline{U}_k$: The structural integrity was unacceptable at k.

The probability of this combined event is,

$$P\left(\overline{U}_k | X_T U_T Y_{T1} D_{41} D_{5k}\right) P(U_T | X_T Y_{T1} D_{41} D_{5k}) P(D_{5k} | X_T Y_{T1} D_{41}) P(D_{41} | X_T Y_{T1}) P(Y_{T1}) P(X_T) = P\left(\overline{U}_k | Y_{T1} D_{5k}\right) P(U_T | X_T) P(D_{5k} | Y_{T1} D_{41}) P(D_{41} | Y_{T1}) P(Y_{T1}) P(X_T) \tag{2.9}$$

Where the first factor is the probability that the integrity is lost in flight k;
The second, the probability of preserved integrity when there is no mechanical damage;

The third, the probability that inflicted damage will grow from region 4 to region 5 during the period flight 1 to flight k;
The fourth, the probability that the size of the accidental damage was in region 4;
Last, the probability that there was no damage at time T.

The effect of production damage only becomes important after it has grown to the size region 4. So the event of interest is: $\overline{X}_T D_{4T} D_{5k} \overline{U}_k$, and the sub-events are:

$\overline{X}_T$: There is damage present at time T;
D_{4T}: The damage size is in region 4 at T;
D_{5k}: The damage size grows to region 5 during the period of flight 1 to flight k;
The integrity is lost at flight k.

The probability of the combined event is,

$$\begin{aligned} &P(\overline{U}_k|\overline{X}_T D_{4T} D_{5k}) P(D_{5k}|\overline{X}_T D_{4T}) P(D_{4T}|\overline{X}_T) P(\overline{X}_T) \\ &= P(\overline{U}_k|D_{5k}) P(D_{5k}|\overline{X}_T D_{4T}) P(D_{4T}|\overline{X}_T) P(\overline{X}_T) \end{aligned} \tag{2.10}$$

Where the factors on the right-hand side are:

The first is the probability of lost integrity at flight k, given damage in region 5;
The second is the probability that the damage grows to region 5 in k flights;
The third is the probability that the damage size is in region 4 at T;
The fourth is the probability that damage will be present at this location at T.

The order of magnitude of the constraint on growth will be determined. In Eq. (2.9) we make the following assumptions for the right-hand side, when $k = n$:

First factor: $0.5 \cdot 10^{-3}$;
Second factor: 0.8;
Third factor: x;
Fourth factor: 10^{-2};
Fifth factor: 10^{-3};
Sixth factor: 0.9.

The value of this contribution to the probability of an unsafe state, $P(\overline{S}_{31}) = 0.45 \cdot 10^{-8} \cdot x$

The value of Eq. (2.10) will be determined using the following assumptions:

First factor: $0.5 \cdot 10^{-3}$;
Second factor: x;

Third factor: 10^{-3};
Fourth factor: 10^{-2}.

The value of this contribution to the probability of an unsafe state, $P(\overline{S}_{32}) = 0.5 \cdot 10^{-8} \cdot x$.

The total contribution to the probability of an unsafe state:

$$P(\overline{S}_{3n}) = 0.95 \cdot 10^{-8} \approx 10^{-8}$$

So, in this case it would represent the "distance" between the lower bound, LB, and the upper bound, UB, in the risk management. This situation (Scenario 3, in Table 2.2) allows some trade-off between the first factor in both contributions and x.

The lower bound for all flights is given by the criterion at T; probability of an unsafe state:

$$P(\overline{S}_T) = P(\overline{H}_\tau \overline{U}_T \overline{H}_T)$$

Which for flight n would yield the lower bound for all scenarios in Table 2.2, and LB is,

$$P(\overline{H}_\tau \overline{U}_T \overline{H}_T \overline{U}_n) = 1 \cdot P(\overline{H}_\tau \overline{U}_T \overline{H}_T) \tag{2.11}$$

Example 2.4: This example deals with scenario 1 in Table 2.2. It includes accidental damage in service and "walk-around" inspections. The first contribution is based on the combined event below: evaluated at the end of the inspection interval. The event is characterized as "there is a damage present at T, the size is in region 4, integrity is not lost, the damage is not detected at T, the damage grows to region 5, is not detected and integrity is lost at flight n,"

$$\overline{S}_{11} = \overline{X}_T D_{4T} U_T \overline{H}_T \overline{H}_{1n} D_{5n} \overline{U}_n$$

where the factors on the right-hand side are

The first: A damage is present at T;
The second: The damage size is in region 4;
The third: The structural integrity is acceptable at T;
The fourth: The damage was not detected at T;
The fifth: The damage was not detected during the n "walk-around" inspections;
The sixth: The damage size has increased in the n flights to belong in region 5;
The seventh: The structural integrity is unacceptable at n.

The probability of the joint event can be expanded as,

$$P(\overline{S}_{11}) = P(U_T|\overline{X}_T D_{4T}\overline{H}_T)P(\overline{U}_n|\overline{H}_{1n}D_{5n})P(\overline{H}_{1n}|\overline{X}_T D_{4T}D_{5n}) \cdot P(D_{5n}|\overline{X}_T D_{4T}\overline{H}_T\overline{H}_{1n}) \cdot P(\overline{H}_T|\overline{X}_T D_{4T})P(D_{4T}|\overline{X}_T)P(\overline{X}_T) \quad (2.12)$$

Where the factors on the right-hand side are in the order: the probability that the integrity was acceptable at T (given appropriate conditions), the probability that the integrity was unacceptable at n, the probability that the damage was not detected in n flights, the probability that the damage grew into region 5 during the n flights, the probability that the damage was not detected at T, the probability that the damage size belonged to region 4 at T, the probability that damage was present at T.

The second contribution describes an accidental damage during operation, $\overline{S}_{12}$ and is a sum of events,

$$\overline{S}_{12k} = X_T U_T Y_k D_{4k} D_{5n}\overline{H}_{kn}\overline{U}_n$$

where the factors on the right-hand side are:

The first factor: Damage is not present at T;
The second: The integrity is acceptable at T;
The third: An accidental damage is inflicted during flight k;
The fourth: The damage size is in region 4 at k;
The fifth: The damage has grown to a size that is in region 5 at flight n;
The sixth: The damage was not detected between flight k and n;
The seventh: The structural integrity was unacceptable at flight n.

The total contribution is,

$$P(\overline{S}_{12}) = \sum_{k=1}^{n} P(\overline{S}_{12k})$$

The expansion of a term in the sum can look like,

$$P(\overline{S}_{12k}) = P(\overline{U}_n|U_T X_T Y_k D_{4k} D_{5n}\overline{H}_{kn})P(U_T|X_T)P(\overline{H}_{kn}|Y_k D_{4k} D_{5n} X_T) \cdot P(D_{5n}|Y_k D_{4k} X_T) \cdot P(D_{4k}|Y_k X_T)P(Y_k)P(X_T) \quad (2.13)$$

where the factors on the right-hand side are: the first, the probability of lost integrity, given a substantial, accidental damage and slow growth; the second, the probability of acceptable integrity, given no damage (if no degradation, this factor would be one); the third, the probability of no detection between flight k and flight n (walk-around); the fourth, the probability of damage growing from region 4 to region 5 in n–k flights; the fifth, the probability that the initial, accidental damage is in region 4; the sixth, the probability of an accidental damage being inflicted at flight k; the seventh, the probability of no damage being present at T.

The third contribution describes damage during maintenance, $\overline{S}_{13}$ and is,

$$\overline{S}_{13} = X_T U_T Y_{T1} D_{41} D_{5n} \overline{H}_{1n} \overline{U}_n$$

where the factors on the right-hand side are:

The first: No damage was present at T;
The second: The structural integrity was acceptable at T;
The third: An accidental damage happened before first flight;
The fourth: The damage size was in region 4 at flight 1;
The fifth: The damage size grew to region 5 by n;
The sixth: The damage was not detected in n "walk-around" inspections;
The seventh: The structural integrity was unacceptable at flight n.

The expansion of the probability of this combined event is,

$$P\left(\overline{S}_{13}\right) = P\left(\overline{U}_n | X_T U_T Y_{T1} D_{41} D_{5n} \overline{H}_{1n}\right) P(U_T | X_T) P\left(\overline{H}_{1n} | Y_{T1} D_{41} D_{5n} X_T\right) \cdot P(D_{5n} | D_{41} Y_{T1} X_T) P(Y_{T1}) P(X_T) \tag{2.14}$$

where the factors on the right-hand side are: the first, the probability of unacceptable damage due to an accidental damage between T and 1 and slow damage growth during the period; the second, the probability of acceptable integrity when no damage is present; the third, the probability of detecting the damage between flight 1 and flight n; the fourth, the probability that the damage will grow from region 4 to 5 during the period; the fifth, the probability that there will be an accidental damage in maintenance $(T, 1)$ the probability that there is no damage present in this location at T.

We have dealt with growth of damage present at T, accidental damage inflicted during operation and its growth, accidental damage during maintenance and its growth, and now we will deal with the situation that is defined by undetected loss of integrity at T.

The fourth contribution is defined by this joint event,

$$\overline{S}_{14} = \overline{X}_T \overline{H}_\tau \overline{U}_T \overline{H}_T \overline{H}_{1n} \overline{U}_n$$

where the factors on the right-hand side are:

The first: Damage is present at T;
The second: No damage was discovered at τ;
The third: Integrity was lost at T;
The fourth: Damage was not discovered by T;
The fifth: Damage was not discovered between flight 1 and n;
The sixth: Integrity was unacceptable at n.

The probability of this combined event can be expanded like this,

$$P(\overline{S}_{14}) = P(\overline{H}_{1n}|\overline{X}_T\overline{U}_T\overline{H}_T\overline{H}_\tau\overline{U}_n)P(\overline{U}_n|\overline{X}_T\overline{U}_T\overline{H}_T\overline{H}_\tau)P(\overline{H}_T|\overline{X}_T\overline{U}_T\overline{H}_\tau) \cdot P(\overline{U}_T|\overline{X}_T\overline{H}_\tau)P(\overline{X}_T|\overline{H}_\tau)P(\overline{H}_\tau) \quad (2.15)$$

where the factors on the right-hand side are: the first, the probability that the damage is not detected in the n "walk-around" inspection at the n flights; the second, the probability that the integrity is unacceptable at n; the third, the probability that damage is not detected at T; the fourth, the probability that integrity is unacceptable at T; the sixth, the probability that damage was present at T; the seventh, the probability that no damage was discovered at τ.

2.3.5. Design constraints summary

We have studied four situations (states at T and types of damages)

- Damage present and size is in region 4, integrity is not lost, damage grows to region 5, not detected and integrity is lost;
- No damage at T, accidental damage in region 4 during operation, damage grows to region 5, not detected and integrity is lost;
- No damage at T, accidental damage between T and 1, damage grows to region 5, damage not detected and integrity is lost;
- Damage present and integrity is lost at T, damage not detected at T, damage not detected during operation and damage either detected or the PSE fails, before n.

Example 2.5: The situations will now be studied numerically. The first situation is described by Eq. (2.12). The factors on the right-hand side are:

The first: 0.5;
The second: y;
The third: $\overline{p}_d^{1n}$;
The fourth: 10^{-5};

The fifth: 0.5;
The sixth: 10^{-2};
The seventh: 10^{-1}.

Here the third factor dominates and is less than 10^{-5} for all practical inspection intervals (>100 flights). The case of interest therefore is when "walk-around" inspections are not possible $\rightarrow p_d = 1$, and the contribution becomes,

$$0.25 \cdot 10^{-6} \cdot y \tag{2.16}$$

where y is the probability that damage grows from region 4 to region 5 in two inspection intervals.

The second situation is described by Eq. (2.13). The factors on the right-hand side are:

The first: 10^{-3};
The second: ~ 1, when no property degradation;
The third: $\overline{p}_d^i$;
The fourth: y';
The fifth: 10^{-1};
The sixth: 10^{-2};
The seventh: 0.9.

The second situation is the product of n factors, and the influences 3 and 4 in each term vary between 1 and n. This effect will be studied in detail in the next example. If we suppose that we concentrate on a limited duration with a reasonable probability of survival, we find that a not unreasonable value for the total factor is 1. The total contribution then becomes,

$$0.9 \cdot y' \cdot 10^{-8} \tag{2.17}$$

and for no access during "walk-around" inspections, it would become $0.9 \cdot y' \cdot 10^{-6}$. This contribution will be explained in a more detailed study in the next example.

The third situation is described by Eq. (2.14). The values of the factors on the right-hand side are:

The first: 10^{-3};
The second: 0.9;
The third: $\overline{p}_d^{1n}$;
The fourth: y;
The fifth: 10^{-3};
The sixth: ~ 0.9.

The total contribution is, $0.8 \cdot 10^{-6} \cdot \overline{p}_d \cdot y$, and for not detecting the damage; the factor describing the effect very quickly becomes less than 10^{-5}, $(0.5^{20} = 10^{-6})$. For the case where a ''walk-around'' inspection is not possible, the contribution is,

$$0.8 \cdot 10^{-6} \cdot y \tag{2.18}$$

The fourth situation is described by Eq. (2.15). The factors on the right-hand side are:

The first: $\overline{p}_d^{1n} \sim 0$, the table illustrates

	n	
$\bar{p}_d$	10	100
0.5	10^{-3}	0
0.4	10^{-4}	0
0.8	10^{-2}	10^{-10}

The second: $\overline{p}_d^{1n}$;
The third: 10^{-3};
The fourth: 10^{-3};
The fifth: ~ 1;
The sixth: 10^{-2}.

The total contribution from the fourth situation thus is,

$$P(\overline{S}_{14}) = 10^{-8} \cdot \overline{p}_d^{1n}$$

which for the case, when the PSE cannot be accessed for ''walk-around'' inspections becomes (assuming growth is considered implicitly),

$$P(\overline{S}_{14}) = 10^{-8}$$

Example 2.6: This example focuses on the effect of accidental damage and growth during the whole inspection period (the effect is measured in terms of probability of growing from region 4 to region 5). The detailed focus is on ''slow damage growth,'' which is defined here as a mean growth that increases moderate damage sizes from region 3 to region 5 in two inspection intervals. The regions are defined as:

$$\text{Region 3: UDD} \le d_s \le \text{GDD};$$

$$\text{Region 5: EDD} \le d_s \le \text{MAD}.$$

Figure 2.11 describes the growth of an arbitrary accidental damage at flight k. The lower limit of integration (LIL) for description of all possible initial accidental

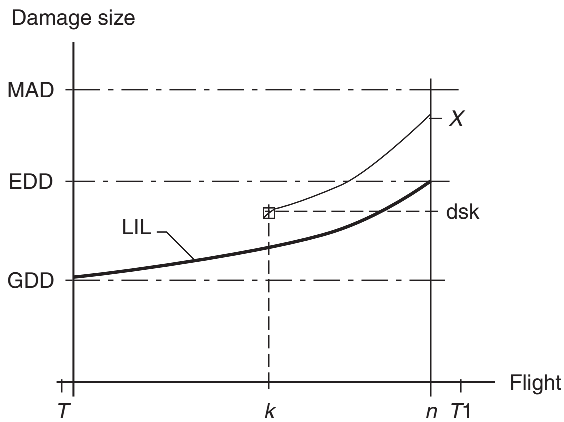

Figure 2.11. Accidental damage at flight k.

damage sizes at a specific flight (if below damage will not grow into region 5 before next inspection).

The highest allowed growth rate (exponential growth limited by the design criterion) is shown in Figure 2.11. The scatter is based on a lowest value of "no-growth" (horizontal line).

The purpose of this example is to illustrate the effect of growth during one period of an accidental damage inflicted during the same period. It is assumed that requirements for damage resistance are such that accidental damage sizes are less than EDD, and the growth requirements preclude reaching region 5 during the inspection period in question. The event of interest is: "Damage growth into region 5, given no damage present at T," and the probability is:

$$P(D_{5n}|X_T) = 1 - P\left(\overline{D}_{5n}|X_T\right)$$

where

$$P\left(\overline{D}_{5n}|X_T\right) = P\left(\overline{D}_{5n}|D_4 Y_{1n} X_T\right) P(D_4|Y_{1n} X_T) P(Y_{1n} X_T)$$

The probability of not growing into region 5 is a product of the probabilities of not growing after an accidental damage at all the flights between 1 and n, Y_{1n}, and for a random flight probability of less than 0.99 we have

	$\bar{y}_{1n}$ for $n = \ldots$		
$\bar{y}_k$	1000	2000	3000
0.99	0	0	0

which clearly yields $y_{1n} = 1$. Studies of both linear growth and exponential growth yield that,

$$\bar{y}_{1n} = \prod_{k=1}^{n} \bar{y}_k \leq \left(\frac{1}{2}\right)^n \to 0, \quad \text{when} \quad (n \to \infty)$$

when slow damage growth has been assured in the structural design. Under these circumstances, we find that the contribution from the second situation is $0.9 \cdot 10^{-6}$ and the choice will stand between improving the damage resistance probability of 10^{-2} to a lower value or the growth rate has to be changed.

The last example shows that the contributions from the four situations in the previous example are, in order,

$$P(\bar{S}_n) = 0.25 \cdot y \cdot \bar{p}_d^{1n} \cdot 10^{-6} + 0.9 \cdot y_{1n} \cdot \bar{p}_d^{i} \cdot 10^{-6} + 0.8 \cdot y \cdot \bar{p}_d^{1n} \cdot 10^{-6} + \bar{p}_d^{i} \cdot 10^{-8}$$

Then factors associated with probability of non-detection are all 1 if no "walk-around" inspection is possible, and the second term is 0 by definition. In that case we find that,

$$P(\bar{S}_n) = y \cdot 10^{-6}$$

and the probability of growing from region 4 to region 5 during one inspection interval should be smaller than $\sim 2 \cdot 10^{-2}$, which means that the lower limit of the growth rate scatter should double the lower damage bound in three inspection intervals.

If the "walk-around" inspection could be done, then the first term would have a small probability of not detecting the damage after m flights

$$P_n = p^m < 10^{-5}$$

and this table illustrates,

p	p^m for $n = \ldots$			
–	5	10	20	100
0.9	0.6	0.35	0.12	$3 \cdot 10^{-5}$
0.7	0.17	0.03	$0.8 \cdot 10^{-3}$	0.000
0.5	0.03	10^{-3}	10^{-6}	0.000

The previous examples show that the following constraints cannot be ignored in the polymeric composites design process, and must be prescribed within pertinent limits and implemented. They are:

Residual strength (with damage and/or degradation, if pertinent);
Damage growth rates;
Damage resistance.

These constraints, applicable to the ultimate and limit load ranges, represent a large part of the foundation of the "Structural Design Criteria."

The role of "walk-around" inspections has also been illustrated. It is shown that the structural safety is dramatically improved, when these inspections are conducted with care and have a prescribed minimum quality. Survival under adverse conditions is affected significantly by these inspections, and they should be part of the detail design considerations. The next example, 2.7, presents a few aspects of safety due to inspection.

Example 2.7: This example focuses on survival, during an inspection interval, of a PSE that enters the service interval with a significant damage (regions 4 or 5). There are, in principle, two situations:

Damage present at T and is not detected;
Severe damage (in region 5) integrity lost and damage not detected.

Figure 2.12 illustrates a definition of damage sizes by probability of detection and the size ranges associated with:

Region 5: MAD $> d_s >$ EDD; expected loss of integrity;
Region 4: EDD $> d_s >$ GDD; size range for severe accidental damage.

It also illustrates an implemented growth requirement that prevents region-4-size-damage to grow larger than region 5 in three inspection periods. Damage size $<$ GDD is prevented from growing into region 4 in one inspection interval. A scatter between the upper limit, UL, growth and the zero-growth state is assumed uniform in the example. Figure 2.12 captures the situation. Upper limit, UL, and lower limit, LL, are shown together with scatter in the left graph, while the right shows the relation between damage size and probability. Size GDD has a probability of $\sim$0.9, in this example.

The probability of survival of one flight is the sum of the probability of detecting the damage during the "walk-around," preflight inspection and the product of the probability of not detecting the damage and the probability of surviving the flight. Figure 2.12 illustrates a conservative way of predicting probability of not surviving

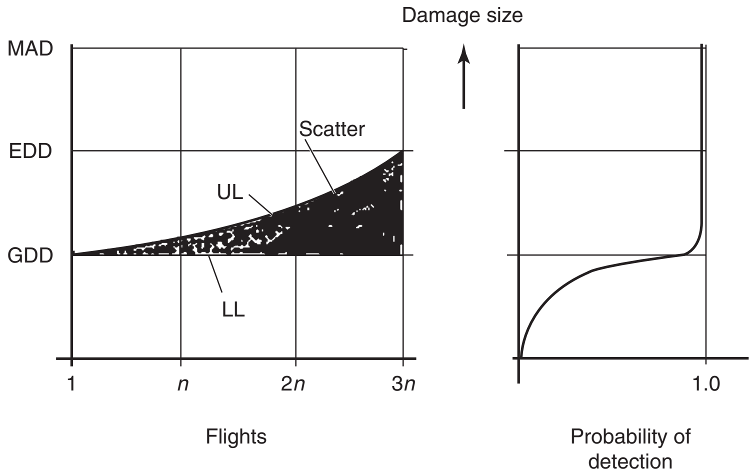

Figure 2.12. Damage growth and probabilities of detection.

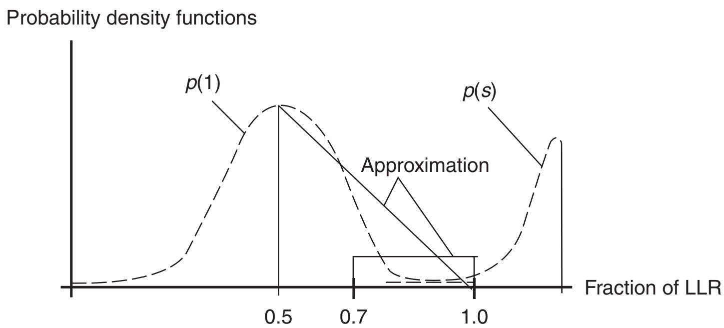

Figure 2.13. Approximation of density functions.

the flight. The density approximation of the residual strength is uniform with a total value in the range (0.7 LLR < RS < LLR) of p_{RS}, and the load approximation is triangular with a total area in the range (0.5 LLR < L < LLR) of p_{L}. Figure 2.13 describes the situation.

This approximation yields

$$\overline{p}_{\mathrm{s}} = 1.96 \cdot p_{\mathrm{L}} \cdot p_{\mathrm{RS}}$$
$$p_{\mathrm{s}} = 1 - \overline{p}_{\mathrm{s}}$$

If we now assume that,

$$p_{\mathrm{L}} = 0.5$$
$$p_{\mathrm{RS}} = 0.01$$

we find that $p_s = 0.99$, and the probability to survive k flights is

$$[p_d + \bar{p}_d\, p_s]^k$$

which for, $p_d = 0.9$, the first case; probability for survival of k flights, yields,

k	k flights with walk-around	k flights, no walk-around
10	0.99	0.9
100	0.90	0.37
1000	0.37	0.0004
2000	0.14	0

suppose $p_s = 0.99999$,

100	0.999	0.99
1000	0.99	0.90
3000	0.97	0.74

Summary: The pervious examples show the importance of detection, residual strength, damage resistance and damage growth to safe composite structural designs. The introduction of design constraints and the development of safety-based design criteria can, e.g. be based on four damage size regions. The following four will be used in the definition of constraints:

$$D_3 \Leftrightarrow \text{MUD} \leq d_s \leq \text{GDD}$$

$$D_4 \Leftrightarrow \text{GDD} \leq d_s \leq \text{EDD}$$

$$D_5 \Leftrightarrow \text{EDD} \leq d_s \leq \text{MAD}$$

$$D_6 \Leftrightarrow \text{MAD} \leq d_s$$

Residual strength safety-based design constraints can be expressed in terms of maximum probabilities for the events,

$$\Pr(\text{RS} < \text{LLR}|\text{D}_i), \quad \text{where} \quad i = 3,\ 4,\ 5$$

Damage resistance constraints can be expressed in terms of the maxima in the following expression,

$$\Pr(D_i|Y), \quad \text{where } i = 3, 4, 5, 6$$

Damage growth rates could be constrained in the design based on,

$$\Pr(D_{in}|D_{j1}), \quad \text{where } j < i,\ i = 3,\ 4,\ 5 \text{ and } j = 4,\ 5,\ 6$$

and 1 and n represent flights.

The application of numerical values to these constraints has to be based on well-defined major inspection programs and quality control of "walk-around preflight" inspections.

A balanced set of design requirements using these types of constraints would be the basis of composite structural safety.

2.4. UNCERTAINTY IN DESIGN

Safety in the traditional "Aluminum World" has a solid foundation in service experience and empirically validated design methods. Not much of that tradition translates well to safe composite structural design. Innovation of structure (especially, for composite structure) means a steady stream of new materials, new processes and new structural concepts. Innovation also means continued introduction of new "better" materials which often makes emerging service experience inapplicable.

Traditional structural design has been focused on the "Extreme Situation" and "worst-case scenarios." Composite structure, critical for damage tolerance and sensitive to operational environment, should be focused on the "Typical Situation" and "Representative Scenarios" to avoid huge costs, long test time and large volumes of data. This could be a practical solution if coupled with uncertainty reduction, risk management, data monitoring and the use of inspection programs as a control process during service.

Tribus (1969) identifies three types of uncertainty as central to Structural Design. They are;

Uncertainty in Data (statistics);
Uncertainty in Hypothesis (model/distribution);
Uncertainty in Knowledge (nature of random phenomena).

In innovation, by its sheer nature, there is uncertainty in requirements. Existing understanding is based on interpretations of results in a "Riveted aluminum skin–stringer world." Practices at airports, repair facilities and depots change both with time and location. Even though Vice President Al Gore's "Commission on Safety in Aviation" collected and analyzed much useful data, it does not translate well into the

World of Composites. The "environment" at airports change; e.g. accidental damage due to impact by construction debris is on the rise. It correlates well with the increase in airport expansions and the fact that change is everywhere and must be considered for safety.

Tribus (1969) and Martz and Waller (1982) deal extensively with the challenge of controlling uncertainty in a rational way.

2.4.1. Uncertainty in residual strength and impact

The situations for damage tolerance composite designs and compliance demonstrations have practically evolved into something very different from the "in-service situation." The following variable types are used in the design and compliance processes.

As shown in Table 2.3, the random nature and the uncertainty of the "in-service environment" is not considered. Table 2.4 describes the variables in service.

Table 2.3. Impact variables in design and compliance

Description		Types of variable		
Variable	Symbol	Random	Deterministic	Fixed
Energy level	e	–	•	Bounded
Impactor radius	r	–	•	•
Damage size	d_s	–	•	–
Location	l	–	•	•
Severity	s	–	•	–
Residual strength	RS	–	•	–
Growth	g	–	•	–
Degradation	d_g	–	•	–

Table 2.4. Impact variables in service

Description		Types of variable			
Variable	Symbol	Random	Deterministic	Uncertain	Fixed
Energy level	e	–	–	•	Unbound
Impactor radius	r	•	–	–	no
Damage size	d_s	•	–	–	no
Location	l	•	–	–	no
Severity	s	•	–	–	no
Residual strength	RS	•	–	–	no
Growth	g	•	–	–	no
Degradation	d_g	•	–	–	no

Important differences can be seen in the comparison between the two tables. The fundamental difference is the many random aspects of the service effects. The most striking example is the fact that in the engineering world a standard spherical (one inch radius) impactor is used for both design data and for compliance. In service, though a variety of impacting objects with a random distribution of the radii in the impacting region are present. The threshold of visibility is often determined by the use of a standard impactor. The determined value is used to set ultimate strength requirements, based on the thesis: "If you cannot see it, you must include the damage in the ultimate strength prediction."

This is of limited value in service, because the number of spherical impactors at work in service is very limited. Instead there exists a random distribution that describes the correlation between external and internal damages. To identify and determine these characteristics is very important for the determination of: "ultimate strength constraints," detection requirements and rules for repair decisions.

Figure 2.14 describes the uncertainties involved in interpreting external damage especially when access is difficult. Figure 2.14 shows a situation with an observed external damage. The measure of damage is presumed to depend on shape, plan-form size, depth and "bluntness" (change in depth over the plan-form) so an interval of uncertainty is shown.

The following probability is the foundation for uncertainties and constraints,

$$P(B_u D_r | D_e) = P(B_u | D_r D_e) \cdot P(D_r | D_e) \tag{2.19}$$

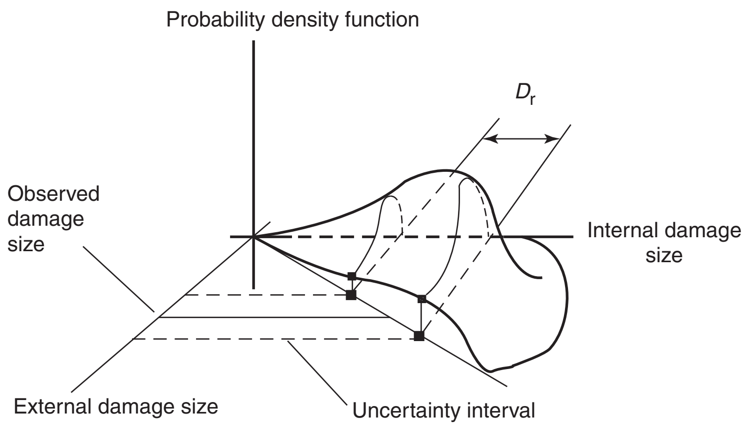

Figure 2.14. Distribution for external and internal damage sizes.

Where the participating events are:

B_u: RS < ULR (ultimate load requirement);
D_r: Range in internal damage size based on uncertainty in external damage sizes;
D_e: External damage size $D_s = d_{s0}$ (best estimate).

The value of the first factor on the right-hand side of Eq. (2.19) can be obtained from "allowables-like" information. The second factor would be obtained from data, like those described in Figure 2.14.

The value of Eq. (2.19) could then be compared to the constraint value required by the design criteria. The analogous approach could be used for LLR, when B_u would be replaced by,

$$B_L : \text{RS} < \text{LLR (limit load requirement).}$$

So between detectability, repair policy and allowables, the uncertainty in residual strength and damage size description can be controlled.

2.4.2. *Uncertainty in damage growth*

Environments and environmentally driven effects have a strong influence on damage growth. Figure 2.15 illustrates one way to recognize uncertainty and incorporate the definition into the design constraints.

Figure 2.15 describes a set of damage sizes that are defined as:

MAD: Maximum Allowed Damage;
EDD: Easily Detectable Damage;
GDD: Good Damage Detectability;
MUD: Maximum Ultimate Damage;

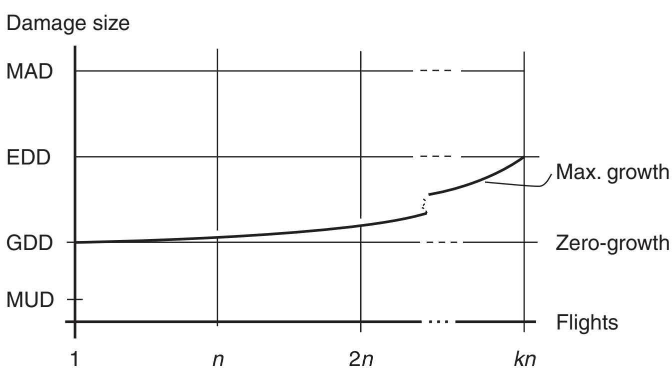

Figure 2.15. Damage growth example.

and n represents the number of flights in a major inspection period.

An objective of controlled maximum growth rate is indicated in Figure 2.15, and here based on the exponential growth that increases damage by L in k inspection periods. Five damage size regions are involved:

Region 2: $d_s < \text{MUD}$;
Region 3: $\text{MUD} < d_s < \text{GDD}$;
Region 4: $\text{MUD} < d_s < \text{EDD}$;
Region 5: $\text{EDD} < d_s < \text{MAD}$;
Region 6: $\text{MAD} < d_s$.

In the description of the unsafe conditions that could exist after a major inspection, and still warrant concerns about survival and damage growth; these regions play an important part. So does controlling growth rate, detectability, residual strength involved in survival and damage resistance. Example 2.8 will illustrate the considerations.

Example 2.8: This example deals with control over three periods, $k = 3$. Exponential growth is used and the following definitions are used,

$$\text{GDD} = L, \ \text{EDD} = 2L, \ \text{MAD} = 3L$$

Scatter is assumed between "no-growth" and "maximum growth" of L in three periods, and the distribution is uniform (initial uncertainty; special knowledge must influence the choice). The probability of unsafe states just after a major inspection is represented by:

$$P\left(\overline{U}_T \overline{X}_T D_{iT} \overline{H}_T\right) = P\left(\overline{H}_T | \overline{U}_T D_{iT} \overline{X}_T\right) \cdot P\left(\overline{U}_T | D_{iT} \overline{X}_T\right) \cdot P\left(D_{iT} | \overline{X}_T\right) \cdot P\left(\overline{X}_T\right) \quad (2.20)$$

which is the probability, at T, of,

- Structural integrity being unacceptable;
- Damage being present;
- Damage size belonging to region i; and
- Damage not being detected.

An assessment of the probabilities of an unsafe state, p_{us}, for three values of i in Eq. (2.20) results in:

$$i = 3 \Rightarrow p_{us} = 10^{-1} \cdot 10^{-5} \cdot 10^{-1} \cdot 10^{-2} = 10^{-9}$$
$$i = 4 \Rightarrow p_{us} = 10^{-2} \cdot 10^{-4} \cdot 10^{-2} \cdot 10^{-2} = 10^{-10}$$
$$i = 5 \Rightarrow p_{us} = 10^{-3} \cdot 10^{-3} \cdot 10^{-3} \cdot 10^{-2} = 10^{-11}$$

The table illustrates orders of magnitude and the importance of damage size regions, showing that both residual strength and detectability should be maintained at pertinent probability levels in the selected regions. However, safety also involves the evaluation of survival under unusual circumstances.

If we now assume that,

$$\overline{T}_T = \overline{U}_T \overline{X}_T D_{iT} \overline{H}_T$$

and that ''walk-around'' inspections at this location are not practical, the probability of survival for three major inspection periods, $K_{1,3n}$, is (p_s is the probability of surviving a random flight)

$$P(K_{1,3n}) = p_s^n \cdot 10^{-1} \cdot p_s^{\mathrm{n}} \cdot 10^{-2} p_s^n = 0.11 \cdot p_s^{3n} \tag{2.21}$$

where a reasonable expectation (supported by Figure 2.15) is that, damage can be in region 3 at T, in region 4 at T_1 and in region 5 at T_2.

Eq. (2.21) applies to a situation when ''walk-around'' inspections cannot practically be done. Supposing a value of $p_s = 0.9$ (not unreasonable for a PSE with lost integrity) we find,

n	$P(K_{1,3n})$ for $p_s = \ldots$				
–	0.9	0.99	0.999	0.9999	0.99999
10	0.005	0.08	0.11	0.11	0.11
100	0	0.005	0.08	0.11	0.11
1000	0	0	0.006	0.08	0.11
3000	0	0	0	0.04	0.10

If we now take a look at a case with ''walk-around'' inspection, any flight and with a probability of detection of 0.8, then we find that

$$P(K_{1,3n}) = 0.11 \cdot p_{cs}^{3n}$$

Which in the specified case yields,

$$P(K_{1,3n}) = 0.11 \cdot 0.98^{3n}$$

Even though it is important to control damage growth and select correct major inspection periods (the main purpose of which must be detecting damage before damage tolerance integrity is lost) a high probability of survival after primary integrity is lost is an important design consideration. This is born out by the success of ''Fail-safe'' structural designs.

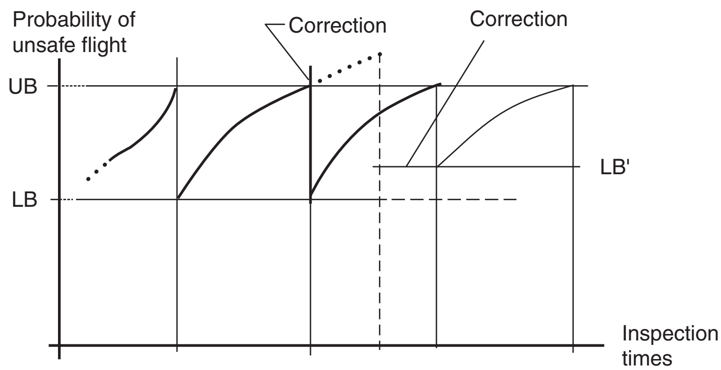

Figure 2.16. Changes in risk during service.

Summary: The requirements on residual strength and detectability in all the regions of damage sizes affect the growth requirement in such a way that the uncertainty in growth characteristics in all environments of importance is quite manageable.

2.5. THE EXTENDED DESIGN PROCESS

The introduction (Chapter 1) shows how structural safety in design can be focused on the premise that safe manufacturing, safe maintenance and safe operation are given. However, it is also shown that safety depends on the inspection quality in such a way that only after a detailed definition of the inspection programs is available, can the "true" detail design constraints and criteria be created.

Innovation often comes with the need for "Design under Uncertainty," which in turn causes requirements for a "Control process" that monitors service and inspection data and reduces uncertainty, updates "a priori" decisions (see Congdon, 2001 and Kullback, 1968) and uses the inspection programs to maintain "Level of Safety" (manages risk). Figure 2.16 illustrates how the risk management, monitoring and updating interacts in service (e.g. due to updating new data).

The "Extended Design Process" that includes consideration of inspection methods, provisions for uncertainty reduction, updating and continued safety monitoring during service would contribute significantly to safety.

Chapter 3
Structural Safety

The safety of composite structures is a challenging subject for many reasons. Not only is it so because innovation by definition precludes substantial use of service data in design methods and in the safety field, but also because service experience accumulates slowly due to the fact that a variety of new materials, new processes and new structural concepts enter the arena continuously.

The successful introduction of composite structures in all applications, where it makes sense, is a very worthy target, but only when the innovations possess better than or equivalent levels of safety compared to the structures they replace.

The ‘‘aluminum design world’’ has produced safe structures for an array of flight vehicles in an environment with an ever-increasing complexity and rising performance demands. We need a new road to success that promises better, safer and cheaper products. The answer that seems to be the most attractive, at this time is, composite structure, if done right. Many successful applications have been introduced, but there still is much more to achieve for commercial airliners, and safety is an important ingredient.

3.1. PRIMARY DRIVERS

Damage tolerance is the dominating safety concern in the design of composite structure, and Chapter 2 provided a number of illustrations of the difficulty in providing survivability after the structural integrity is lost, especially for long inspection periods.

It seems that an alternative strategy would be to make ‘‘undetected loss of integrity’’ a very rare event. So the event of interest is a loss of integrity that eludes detection at a major inspection. The following events are involved:

$\overline{H}_T$: Damage was not detected at T;
$\overline{U}_T$: Integrity is unacceptable at T;
$\overline{X}_T$: Damage is present at T;
Y_{T1}: An accidental damage was inflicted before first flight after an inspection at T;
T_T: The total situation before the first flight after an inspection.

The probability of an unacceptable level of integrity, and undetected damage for the first flight after a major inspection is,

$$P(T_T)=P(\overline{U}_{T1}\overline{X}_T\overline{H}_T) \tag{3.1}$$

We assume that during the time dedicated to inspection, repair and general maintenance (between T and 1) an accidental damage can occur, even though the PSE cannot be impacted during normal operation. Eq. (3.1) can be expanded as,

$$P(T_T)=P(\overline{U}_T\overline{X}_T\overline{H}_T\overline{H}_\tau)+P(U_TX_TY_{T1}\overline{U}_1) \tag{3.2}$$

Suppose that the regions 4 and 5 are the only two being involved (safe maintenance is given) then the following expansion results,

$$P(T_T)=P(\overline{U}_T\overline{X}_T\overline{H}_T\overline{H}_\tau D_{3T})+P(\overline{U}_T\overline{X}_T\overline{H}_T\overline{H}_\tau D_{4T})+P(U_TX_TY_{T1}\overline{U}_1) \tag{3.3}$$

where τ is the inspection, just prior to the one at T. The first term on the right-hand side of Eq. (3.3) represents the probability of: "the integrity being unacceptable, damage being present and not detected at, nor at the previous inspection at and the damage belongs to region 3."

The second term is analogous, except, the damage belongs to region 4. The third term represents the probability of: "the integrity being acceptable at T, damage not being present, an accidental damage occurring between T and 1 and integrity being unacceptable at the start of the first flight."

Example 3.1: This example uses Eq. (3.3) to illustrate orders of magnitude of probabilities involved in the, "undetected loss of integrity."

An expansion further of the terms in the right-hand side of Eq. (3.3) yields, for the first,

$$P(\overline{U}_T\overline{X}_T\overline{H}_T\overline{H}_\tau D_{5T})=P(\overline{H}_T|\overline{U}_T\overline{X}_TD_{5T}\overline{H}_\tau)P(\overline{U}_T|\overline{X}_TD_{5T}\overline{H}_\tau)P(D_{5T}|\overline{X}_T\overline{H}_\tau)$$
$$P(\overline{X}_T|\overline{H}_\tau)P(\overline{H}_\tau)$$

The first term $P(T_{1T})$ can the be assessed,

$$P(T_{1T})=10^{-3}\cdot10^{-2}\cdot10^{-1}\cdot10^{-1}\cdot10^{-2}=10^{-9}$$

And for the second term (analogously)

$$P(T_{2T})=10^{-2}\cdot10^{-3}\cdot10^{-1}\cdot10^{-3}=10^{-9}$$

And finally, the third term,

$$P(T_{3T}) = P\left(U_T X_T Y_{T1} \overline{U}_1\right) = P\left(\overline{U}_1 | X_T Y_{T1} U_T\right) P(U_T | Y_{T1} X_T) P(Y_{T1} | X_T) P(X_T)$$

For which the following estimate shows order of magnitude

$$P(T_{3T}) = 10^{-4} \cdot (\sim 1) \cdot 10^{-3} \cdot 10^{-2} = 10^{-9}$$

The first and second terms of Eq. (3.3) are "driven by detectability, damage tolerance and damage resistance," and the third by damage tolerance and damage resistance. The purpose of this example and its estimates is to show the influence of the major safety drivers and how they relate to the design constraints. The first two terms on the right-hand side of Eq. (3.3) could provide the basis for the lower bound (value after a major inspection) of the risk.

3.2. RISK MANAGEMENT

Figure 3.1 illustrates a risk management with control by inspection. For a given inspection method, the length of the inspection period or the inspection approach can be used to limit the increase in risk between inspections.

The increase in risk between inspections can be due to:

Accidental damage during the inspection–maintenance activity (between T and 1) and following joint events describe the total situation:

$$R_1 = U_T X_T Y_{T1} D_{i1} D_{jk} \overline{U}_k \overline{H}_{1k}$$

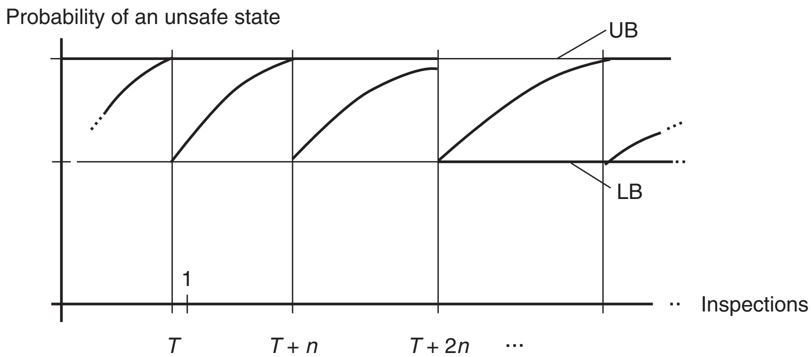

Figure 3.1. Risk management by inspection.

which represents: acceptable integrity at T, no damage present at T, accidental damage between T and 1, initial damage in region i ($i = 2, 3, 4$), damage grows to region j in k flights, integrity is lost by flight k and the damage is not detected between flight 1 and k.

The probability is:

$$P(R_1) = P\left(\overline{U}_k | D_{jk} \overline{H}_{1k} Y_{T1} D_{i1} X_T U_T\right) \cdot P\left(\overline{H}_{1k} | D_{jk} Y_{T1} D_{i1} U_T X_T\right) \cdot P\left(D_{jk} | Y_{T1} D_{i1} U_T X_T\right) \cdot$$
$$P(D_{i1} | Y_{T1} U_T X_T) \cdot P(Y_{T1} | U_T X_T) \cdot P(U_T | X_T) \cdot P(X_T) \tag{3.4}$$

1. Damage growth between 1 and k is defined by this situation:

$$R_2 = U_T \overline{X}_T D_{iT} D_{jk} \overline{H}_{1k} \overline{U}_k$$

which represents: acceptable integrity at T, damage present at T, damage size in region i ($i = 2, 3, 4$) at T, damage growth to region j in k flights, damage not detected between 1 and k and integrity unacceptable at k.

The probability is:

$$P(R_2) = P\left(\overline{U}_k | D_{jk} D_{i1} \overline{H}_{ik} \overline{X}_T U_T\right) \cdot P\left(\overline{H}_{1k} | D_{jk} D_{i1} \overline{X}_T U_T\right) \cdot$$
$$P\left(D_{jk} | D_{i1} \overline{X}_T U_T\right) \cdot P\left(D_{i1} | \overline{X}_T U_T\right) \cdot P(U_T | X_T) \cdot P(X_T) \tag{3.5}$$

2. Degradation is defined by:

$$R_3 = U_T X_T X_k \overline{U}_k$$

which represents: acceptable integrity at T, no damage present at T, no mechanical damage present at k and integrity unacceptable at k.

The probability is:

$$P(R_3) = P\left(\overline{U}_k | X_k X_T U_T\right) \cdot P(X_k | X_T U_T) \cdot P(U_T | X_T) \cdot P(X_T) \tag{3.6}$$

3. Accidental damage at m with growth to k, one term in the sum is,

$$R_{4m} = U_T X_T Y_m D_{im} D_{jk} \overline{H}_{mk}$$

which represents: acceptable integrity at T, no damage present at T, accidental damage inflicted at m, initial damage size is in region i, damage size is in region j at k.

The probability is:

$$P(R_{4m}) = P\big(\overline{U}_{k|}|D_{jk}D_{im}Y_m\overline{H}_{mk}U_TX_T\big)\cdot P\big(\overline{H}_T|D_{jk}D_{im}Y_mU_TX_T\big)\cdot$$
$$P(D_{jk}|D_{im}Y_mU_TX_T)\cdot P(D_{im}|Y_mU_TX_T)\cdot$$
$$P(Y_m|U_TX_T)\cdot P(U_T|X_T)\cdot P(X_T) \tag{3.7}$$

where these probabilities can be used to assess the increase in risk.

Example 3.2: The purpose of this example is to indicate orders of magnitude of the probabilities in Eq. (3.4)–(3.7). The "growth" is described in Figure 3.2.

Applying the growth in Figure 3.2 considering the following alternatives:

Initial region	Final region
3	3
3	4
4	4
4	5

and assuming that "walk-around" inspections are impractical for this location, degradation is negligible and it is protected from accidental damage during operation, we make the following assessments (it is assumed that integrity is lost when

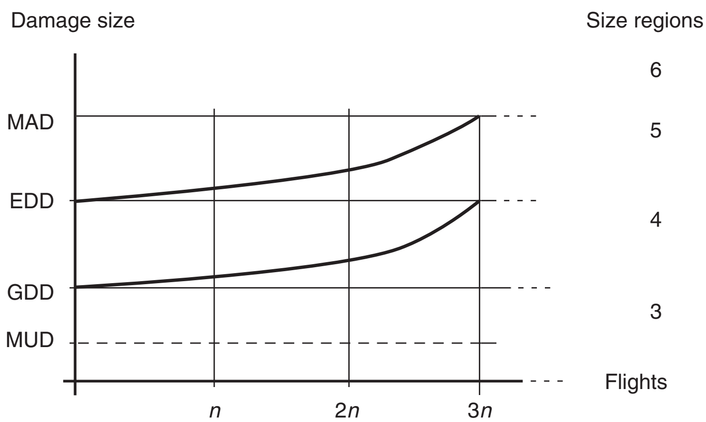

Figure 3.2. Damage growth and size regions.

RS < LLR):

$$P(R_{11}) = 10^{-7} \cdot 1 \cdot 0.7 \cdot 10^{-1} \cdot 10^{-2} = 7 \cdot 10^{-11}$$
$$P(R_{12}) = 10^{-5} \cdot 1 \cdot 0.3 \cdot 10^{-1} \cdot 10^{-2} = 3 \cdot 10^{-9}$$
$$P(R_{13}) = 10^{-5} \cdot 1 \cdot 0.7 \cdot 10^{-1} \cdot 10^{-2} = 7 \cdot 10^{-9}$$
$$P(R_{14}) = 10^{-3} \cdot 1 \cdot 0.3 \cdot 10^{-3} \cdot 10^{-2} = 3 \cdot 10^{-9}$$

The total probability becomes

$$P(R_1) \approx 1.3 \cdot 10^{-8}$$

A study of Eq. (3.4) reveals that, $P(R_2) \approx P(R_1)$, and the total probability increase, during one inspection interval, of the risk in this example is:

$$\Delta P(R) = 2.6 \cdot 10^{-8}$$

This example is an llustration of how important residual strength and detection is for the risk and how it influences the design constraints. Finally, for an LB of 10^{-8}, we would have an UB of $3.6 \cdot 10^{-8}$.

When considering the values selected for the first factor on the right-hand side of the evaluated expressions, it is interesting to notice the progression of the values for the probability of not meeting limit requirements in different regions,

$$P(\overline{B}|D_3) = 10^{-7}$$
$$P(\overline{B}|D_4) = 10^{-5}$$
$$P(\overline{B}|D_5) = 10^{-3}$$

Where $\overline{B}$ is the event "residual strength is less than LLR," and D_i is the event "damage size belongs in region i."

If one compares these values with modern "allowables-values," then for,

$$B\text{-values: } \Pr(S < F_B) = 0.10, \quad \text{and for}$$
$$A\text{-values: } \Pr(S < F_A) = 0.01$$

This suggests that a different balance between "Damage tolerance" and "Damage resistance" could be desirable, because a value for,

$$P(\overline{B}|D_5) = 10^{-3}$$

is very difficult to achieve. The balance could be based on,

$$P(\overline{B}|D_5)\cdot P(D_{41}\,|\,Y_{T1}) = 10^{-6}$$

and, e.g. produce $P(\overline{B}|D_5) = 10^{-2}$ and $P(D_{41}\,|\,Y_{T1}$ or $\overline{X}_T) = 10^{-4}$, again demonstrating the complexity in setting design requirements and managing risk.

3.3. IMPORTANCE OF SAFETY REGULATIONS

Airworthiness regulations like FAR and JAR 25 (and the supporting documentation) have for a long time had a large influence on safety requirements, and engineering practices. In many situations, it has taken on characteristics as minimum standards, and left interpretation to the practicing community. For example "limit load" is defined as:

"The largest load expected in service."

An interpretation that eliminates uncertainty is necessary to be able to design damage tolerance critical structure. This chapter discusses extending the existing and emerging regulations (advisory circulars, ACs), so that states of uncertainty, randomness and knowledge can be considered in a rational way in the design process.

3.3.1. Limit load regulations

Figure 3.3 shows a possible definition that is specific to a PSE and a location, where the internal loads are produced by required loads envelopes.

The requirement of limit load capability could be interpreted as satisfying the most critical loading (including interaction) "limit load," if the internal loads are the basis for limit loads requirements and the most critical situation is considered to be:

$$L_{\max} = \max\left\{\vec{N}_{\max\ i}\right\};$$

for all flights (flight types) and for tension, compression and shear dominated internal loads situations. The associated probability statement, then could be:

"The probability that the largest loading at a specific location of a PSE, 'limit load,' will occur during a 'lifetime' of the PSE is one."

$$\Pr(L_{\max} > \mathrm{LLR}|O) = 0$$

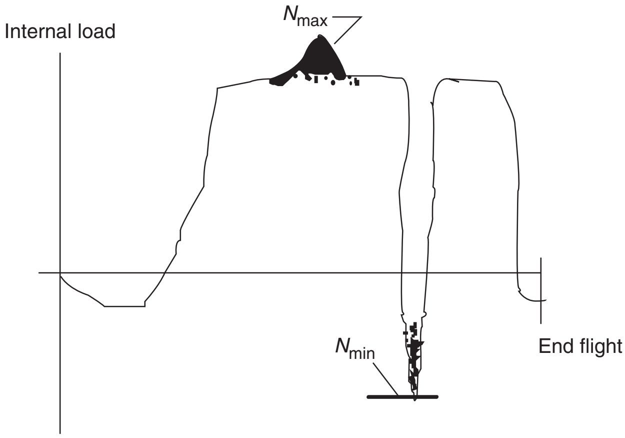

Figure 3.3. Internal loads during flight; a specific location.

when considering the results from Chapter 1. This extension of the definition of limit load and limit load capability makes it possible to implement safety-based damage tolerance design and sizing.

3.3.2. Allowables regulations

Among others, FAR 25 states that for multiple load path structure (fail-safe structure) *B*-value allowables can be used. *B*-value allowables are defined as the strength value exceeded by a probability of 90 per cent with 95 per cent confidence. Example 3.3 demonstrates a basis for probability requirements for residual strength in different regions of damage sizes, based on fail-safety.

Example 3.3: We will start by assuming normally distributed allowables, recognizing that for composites a damage definition is required even for ultimate strength. We also assume, in accordance with FAR 25 that the ultimate safety factor is 1.5. Figure 3.4 shows the damage regions we will study and use for expressing the relation between average damage size and the residual strength mean for potential residual strength probability distributions.

We now assume that the ultimate strength region includes regions 1 and 2, and that the use of *B*-values results in:

$$\Pr(\mathrm{RS} \leq F_B | D_{12}) = 0.10 \Rightarrow \Phi(t) = 0.10 \quad \text{and}$$
$$t = \frac{x - \mu}{\sigma} = -1.30 \quad \text{and}$$
$$x = F_B = \mu(1 - 1.30 \cdot C_{\mathrm{v}})$$

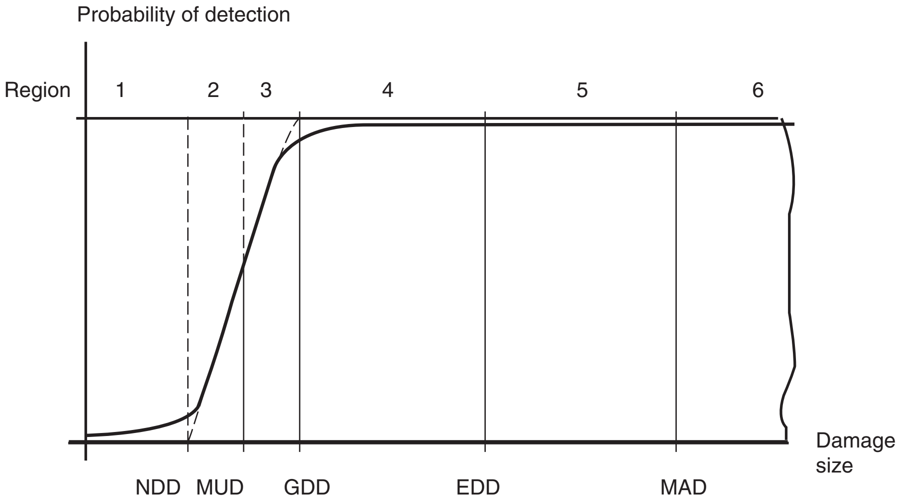

Figure 3.4. Damage sizes and regions.

where $\Phi(t)$ represents the standardized normal distribution, and the probability of the "limit value," $F_B/1.5$, for linear structures can be expressed as,

$$\text{with} \quad t_1 = \frac{((F_B/1.5) - \mu)}{\sigma} \text{ as } \Phi(t_1) \quad \text{and} \quad t_1 = -\frac{1}{3C_v} - 0.87$$

resulting in the following table:

C_v	t_1	$\Phi(t_1)$
0.05	−7.54	0
0.10	−4.20	10^{-5}
0.15	−3.09	10^{-3}

An optimistic evaluation could then lead to the assessment:

$$\Pr(\text{RS} < \text{LLR}|D_1) = 10^{-5}$$

If we continue into region 4, we have with $\mu_{4\text{RS}} = k_4 \cdot (F_B/0.87)$ the following,

$$t_4 = \frac{(0.667 - (k_4/0.87)) \cdot 1.75}{C_v}$$

and the table in region 4 becomes,

k_4	t_4 for $C_v = \ldots$			$\Phi(t_4)$ for $C_v = \ldots$		
–	0.10	0.15	0.20	0.10	0.15	0.20
0.8	−4.42	−2.95	−2.21	$4 \cdot 10^{-6}$	10^{-3}	0.014
0.9	−6.43	−4.28	−3.22	0	10^{-5}	0.0006

We now return to the assumption of the reduction of the mean of residual strength being a function of the square root of the ratio of the appropriate damage sizes, we find that the conclusion is

$$k_4 = 0.82$$

and a reasonable value for region 4 could be,

$$\Pr(\mathrm{RS} \leq \mathrm{LLR}|D_4) = 10^{-4}$$

An analogous argument for region 5 would lead to this table,

k_5	t_5 for $C_v = \ldots$			$\Phi(t_4)$ for $C_v = \ldots$		
–	0.10	0.15	0.20	0.10	0.15	0.20
0.6	−0.396	−0.03	−0.02	0.34	~0.5	~0.5
0.7	−2.41	−1.61	−1.20	0.008	0.05	0.11

which for $k_5 = 0.63$ can be estimated as,

$$\Pr(\mathrm{RS} \leq \mathrm{LLR}|D_5) = 10^{-2}$$

The purpose of this example is to analyze the possibilities and practical orders of magnitude that apply to the requirements and the constraints in the structural design process. The emerging "picture" indicates that very close attention has to be paid to the balance between safety and practicality in order to produce realistic requirements, but it also indicates that ignoring the random nature of structural damage, environment and operating variations paint an erroneous safety picture.

3.4. UNCERTAINTY, PROBABILITY AND STATISTICS OF DAMAGE TOLERANCE

The centerpiece of damage tolerance, in structural design, is residual strength, and randomness of the variables involved is very important for structural safety.

The primary random variables affecting residual strength are:

- Damage size (internal), d_s;
- Damage severity, s_e;
- Property variability d_g,

but also because of the safety value of detection,

- Detectability (detection $= H$);
- External damage size, d_e.

So it is possible to express the Probability of an "Unsafe state," at time T, as,

$$P(\overline{S}_T) = P(\overline{U}_T\overline{H}_T) = P(\overline{U}_T\overline{X}_T\overline{E}_T\overline{H}_T D_l) + P(\overline{U}_T\overline{X}_T\overline{E}_T\overline{H}_T\overline{D}_l) + P(\overline{U}_T\overline{X}_T E_T\overline{H}_T D_l) \quad (3.8)$$

where

B: RS > LLR, and $\overline{B}$ is the major influence on $\overline{U}_T$
E: External damage not present;
X: Internal damage not present;
H: Damage detected.

And the following regions are involved,

$$D_l = D_1 \cup D_2 \cup \cdots \cup D_5 \quad (3.9)$$

The first term on the right-hand side of Eq. (3.8) can be expanded to

$$P(\overline{U}_T|\overline{H}_T\overline{X}_T\overline{E}_T D_l) \cdot P(\overline{H}_T|\overline{X}_T\overline{E}_T D_l) \cdot P(D_l|\overline{X}_T\overline{E}_T) \cdot P(\overline{X}_T|\overline{E}_T) \cdot P(\overline{E}_T) \quad (3.10)$$

The second term is ~0. The third term can be expanded to,

$$P(\overline{U}_T|\overline{H}_T\overline{X}_T E_T D_l) \cdot P(\overline{H}_T|D_l\overline{X}_T E_T) \cdot P(D_l|\overline{X}_T E_T) \cdot P(\overline{X}_T|E_T) \cdot P(E_T) \quad (3.11)$$

and we could write the expansion of Eq. (3.10) for one of the subsets in Eq. (3.9) in the following form,

$$P(\overline{U}_T\overline{X}_T\overline{E}_T D_{iT}\overline{H}_T) = P(\overline{U}_T|\overline{H}_T\overline{X}_T\overline{E}_T D_{iT}) \cdot P(\overline{H}_T|D_{iT}\overline{X}_T\overline{E}_T) \cdot P(D_{iT}|\overline{X}_T\overline{E}_T) \cdot P(\overline{X}_T|\overline{E}_T) \cdot P(\overline{E}_T), \text{ where } i = 1, \ldots, 5 \quad (3.12)$$

The expansion of Eq. (3.11) would be analogous, and the only difference would be that,

$$\overline{E}_T \text{ would be replaced by } E_T$$

Example 3.4: This example deals with an order of magnitude investigation of Eq. (3.10) for the purpose of illustrating the practicalities of the different probabilities.

The first factor of Eq. (3.10) can be expected for $i=4$ to be $\sim 10^{-4}$ and for $i=5$, $\sim 10^{-2}$. The second, for $i=4$, ~ 0.05 and for $i=5$, $\sim 10^{-2}$.

The five factors on the right-hand side of Eq. (3.10) are all based on different states of uncertainty, knowledge and confidence.

So for Eq. (3.10), totally, for $i=5$: $10^{-2} \cdot 10^{-2} \cdot 10^{-3} \cdot 10^{-2} = 10^{-9}$ and sum would be 10^{-9}.

3.4.1. Uncertainty in damage

In contrast to the well organized, orderly world of "testing for characterization and compliance" the service environment presents a number of complications, such as contact areas that vary from sharp corners to flat surfaces. Spherical impactors, especially those with one inch radius, are not well represented in service. As a consequence, we find external damage from the very localized imprints with deep indentations to large surface areas with next to unnoticeable depths.

In many situations we find that external damage is the only one available for both detection and assessment of residual strength. In a perfect world there would be a unified measure that would totally describe both size and severity of internal damage leading directly to the residual strength of the structure in question. Figure 3.5 illustrates the point.

However, in reality there are probabilistic relationships between the variables. Examples of the joint probability density functions can be found in Figures 3.5 and 3.6.

The uncertainty is emphasized by the fact that a descriptive measure of external damage at least must contain assessments of: size, depth and characteristics in order to be an effective representation.

3.4.2. Uncertainty in detection

The first term in Eq. (3.8) deals with detection. The first factor in the right-hand side of Eq. (3.10) and the analogous factor in the expansion of the third term of Eq. (3.8) also deals with detection. An attempt to describe the total event that involves both

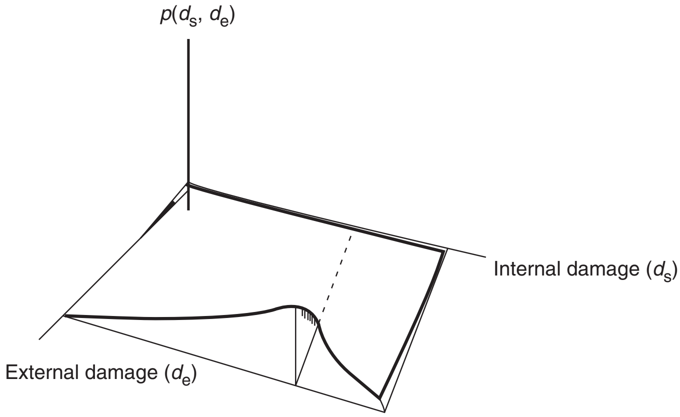

Figure 3.5. Damage joint probability density.

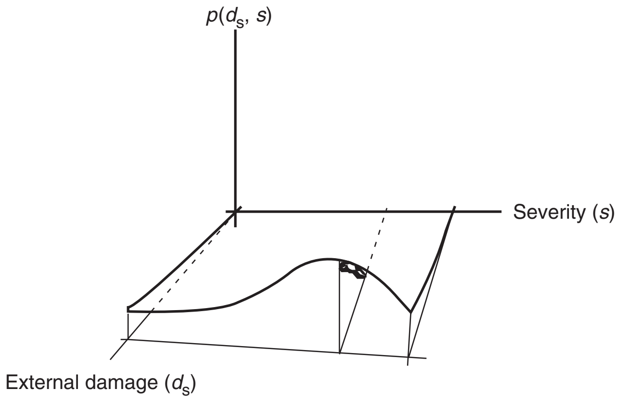

Figure 3.6. Joint probability density for external damage and severity.

residual strength and the total state of damage and detection at time T could involve the following sub-events:

B_T: RS > LLR;
T_T: The combined event describing the state of damage and detection;
$\overline{X}_T$: Internal damage present at time T;
D_{iT}: Internal damage size is in region i at T;
$\overline{E}_T$: External damage present at T;
Z_{jT}: External damage is in region j at T;
F_{mT}: Severity factor is in region m at T;
H_T: Damage detected at T.

The probability of an "acceptable integrity and a specific state of damage and detection" can be written as:

$$P(B_T T_T) = P(B_T|T_T) \cdot P\big(F_{mT}|\overline{X}_T D_{iT} \overline{E}_T Z_{jT} H_T\big) \cdot \\ P\big(H_T|\overline{X}_T D_{iT} \overline{E}_T Z_{jT}\big) \cdot P\big(\overline{X}_T D_{iT} \overline{E}_T Z_{jT}\big) \tag{3.13}$$

The second factor of the right-hand side of Eq. (3.13) is a statement about the probability of severity being in the region m, where $m = 1, \ldots, 5$. The third factor is the probability of detection, given a certain state of external and internal damage.

The last factor deals with the probability of the state of external and internal damage. Here is where the uncertainty of the "true reality" enters. An expansion of the factor will be discussed. The basis will be the following definition of external damage size regions:

Z_{0T}: The external damage size, $d_e = 0$;
Z_{1T}: $0 < d_e <$ BVID (barely visible damage);
Z_{2T}: BVID $< d_e <$ CVD (clearly visible damage);
Z_{3T}: CVD $< d_e <$ EDD (easily detectable damage).

We write the state of damage, S_D as,

$$P(S_D) = P\big(D_{iT}|Z_{jT}\overline{X}_T\overline{E}_T\big) \cdot P\big(\overline{X}_T|\overline{E}_T Z_{jT}\big) \cdot P\big(Z_{jT}|\overline{E}_T\big) \cdot P\big(\overline{E}_T\big) \tag{3.14}$$

Suppose we focus on large internal damage, region $i = 5$,

$$P(S_D) = \sum_{j=1}^{3} \big[P\big(D_{5T}|Z_{jT}\overline{X}_T\overline{E}_T\big) \cdot P\big(\overline{X}_T|\overline{E}_T Z_{jT}\big) \cdot P\big(Z_{jT}|\overline{E}_T\big) \cdot P\big(\overline{E}_T\big)\big] \tag{3.15}$$

The case of no external damage could then be expressed as,

$$P(S_D) = P\big(D_{5T}|\overline{X}_T E_T\big) \tag{3.16}$$

and for a special case Eq. (3.15) reduces to one term (e.g. $j = 2$). We are now faced with three different situations identifying the uncertainty:

- The two damages are directly related through a recent event;
- The event started as an impact, but considerable growth has made correlation in size very difficult;
- There never was an external damage. The internal damage grew from a manufacturing flaw.

The uncertainty in detection is closely tied to the uncertainty in damage size and correlation. The situation requires an "a priori" probability assessment based on the importance of different damage sizes for different inspection methods.

Again, monitoring of emerging service data is of great importance for uncertainty reduction and risk management in service.

3.4.3. Uncertainty of residual strength

Residual strength can be visualized as a function of d_s, s and p, where,

p represents material properties;
s severity; and
d_s internal damage size,

and all are random variables. The relation can be expressed as,

$$\text{RS} = \text{RS}[d_s(t), s(t), p(t)] \tag{3.17}$$

If we now focus on the key event in loss of integrity,

$$\overline{B}_T\text{: RS} \leq \text{LLR}$$

and extend the total stage of damage, detection and degradation to include the change in scatter of properties, T'_T could be defined as,

$$T'_T = P_T D_{iT} F_{mT} Z_{jT} \overline{X}_T \overline{E}_T \tag{3.18}$$

Where P_T could represent the event,

$$P_T : \text{Strain energy rates are less than } \vec{G},$$

and $P(\overline{B}_T | T'_T)$ would mean the probability that,

"RS < LLR, given internal damage in region i, external damage in region j and severity in zone m"; $i = 4, 5,\ j = 0, 1, \ldots,\ 3$ and $m = 1, 2, 3$.

The probability can also be thought of as the probability that,

$$\text{RS} = \text{RS}[d_s(t),\ s(t),\ p(t)] \leq\ \text{LLR} \tag{3.19}$$

given the total state T'_T. The uncertainty associated with residual strength can be considered as represented by the random nature of the relation expressed by

Eq. (3.19), which alternatively, depending on the "composite material," could be written as,

$$\mathrm{RS} = \mathrm{RS}_0(d_s,\ s) \cdot f(p)$$

or (3.20)

$$\mathrm{RS} = g(d_s) \cdot h(s) \cdot f(p)$$

both implicitly functions of t, time.

The uncertainty also pertains to the probability model; the number of parameters, the nature of the distributions and the practical range of the variables,

$$0 \leq d_s \leq \text{MAD?}$$
$$0 \leq s \leq 1?$$
$$0 \leq p \leq \text{SER?}$$

The uncertainty in statistical confidence can be characterized by the following three questions:

- How much data?
- What level of data; coupon, element, etc.;
- What quality of scale-up? (e.g. from element to sub-component).

The challenge in "probability" is contained in the choice between postulating a priori probability distributions or assigning probability levels of intervals for damage and severity. Intervals for damage would be a natural choice if an "allowables approach" was used.

The following equation deals with the probability of loss of limit load residual strength capability, and is the basis for design data definition and a large part of the design criteria formulation,

$$P\left(\overline{B}_T T'_T\right) = P\left(\overline{B}_T | T'_T\right) \cdot P\left(F_{mT} | D_{it}\overline{X}_T Z_{jT}\overline{E}_T P_T\right) \cdot P\left(D_{iT}\overline{X}_T Z_{jT}\overline{E}_T P_T\right) \qquad (3.21)$$

The second factor on the right-hand side of Eq. (3.21) makes a probability statement about "severity" and can be used for choosing "reasonable" regions of severity for designing the structure. The uncertainty of severity can be traced back to, among others, mixture and type of damage. A number of types are involved,

- Fiber breakages;
- Delaminations;
- Debonds;
- Matrix cracking;
- Nature of damage front, etc.

to mention a few. The third factor makes a statement about damage. A very typical damage in the "aluminum world" is through thickness cracks. However, in the composite world most damage has several dimensions, and can often be described as having external and internal characteristics. The third factor in Eq. (3.21) can be expanded as,

$$P\left(D_{iT}|\overline{X}_T\overline{E}_T Z_{jT} P_T\right) \cdot P\left(\overline{X}_T|\overline{E}_T Z_{jT} P_T\right) \cdot P\left(Z_{jT}|\overline{E}_T P_T\right) \cdot P\left(\overline{E}_T|P_T\right) \cdot P(P_T) \qquad (3.22)$$

The first factor can be interpreted as "the probability that the internal damage size is in region *i*, when both internal and external damage is given, external damage size is in region *j* and the property scatter is given by P_T."

The main uncertainty in this factor is due to an ever-changing "service environment," presenting everything from fork-lifts to hailstones, but very few "spherical impactors." This represents an uncertainty that should be covered by postulates for the design criteria formulation and subject to monitoring and updating in service. The second factor in expression (3.22) could be considered to deal with: "the probability that there is an internal damage, given the existence of a specific external damage and a given property state."

Eq. (3.21) describes a recent accidental damage. There is an analogous case for damage that has grown into its present size. The probability of that situation can be expressed as,

$$P\left(\overline{B}_T D_{iT}\overline{X}_T F_{mT} P_T\right) = P\left(\overline{B}_T|D_{iT}\overline{X}_T F_{mT} P_T\right) \cdot P\left(F_{mT}|D_{iT}\overline{X}_T P_T\right) \cdot P\left(D_{iT}\overline{X}_T P_T\right) \quad (3.23)$$

where the uncertainty associated with the second factor on the right-hand side describes a very difficult situation, because of a vague definition of the causing factors and little opportunity to gain more insight through monitoring in service.

A comparison between Eqs. (3.21) and (3.23) reveals another uncertainty. That uncertainty is associated with damage initiated as accidental damage, but which has grown to its "present" size. The external damage is not representative of the actual internal damage, either because of the time element by itself or in combination with changes in external size due to relaxation in the resin.

So, the uncertainty influences both predictions of internal damage size and severity. It is reasonable for this uncertain situation to deal with damage size and severity as criteria, and detectability as only marginally related to external damage. The third factor of the right-hand side Eq. (3.13),

$$P\left(H_T|\overline{X}_T D_{iT}\overline{E}_T Z_{jT}\right)$$

should be evaluated in combination with the three different states of compatibility,

M: External and internal damage is in a one-to-one correspondence;
$\overline{M}$: External and internal damage is independent;
N: No external damage present,

which could result in the following equation:

$$P\left(\overline{B}_T T'_T\right) = P\left(\overline{B}_T T'_T M\right) + P\left(\overline{B}_T T'_T \overline{M}\right) + P\left(\overline{B}_T T'_T N\right) \tag{3.24}$$

where the first term represents Eq. (3.13) and the third factor in Eq. (3.23). The second leads to,

$$P\left(\overline{B}_T T'_T \overline{M}\right) = P\left(\overline{B}_T | T'_T \overline{M}\right) \cdot P\left(F_{mT} | \overline{X}_T D_{iT} \overline{M} H_T\right) \cdot P\left(H_T | \overline{X}_T D_{iT} \overline{M}\right) \cdot$$
$$P\left(\overline{M} | \overline{X}_T D_{iT}\right) \cdot P\left(\overline{X}_T D_{it}\right) \tag{3.25}$$

And the major uncertainty resides with severity and next to the last factor in Eq. (3.25).

3.4.4. Monitoring and updating

Monitoring of service data (including inspection results) has two objectives,

- Reducing uncertainty;
- Providing data to update the probability base.

The bulk of service data deals with damage sizes, damage locations, damage types and damage characteristics. The primary aims of the monitoring are to reduce the uncertainty about the probability of damage in specific locations, to test the used probability density functions (or probabilities of damage sizes in different size regions) used in design, and, if necessary to update the a priori information by using methods like Bayesian updating (see Stirzaker, 2003).

Figures 3.7–3.9 illustrate the kind of information that would be valuable in achieving the above objectives.

Figure 3.7 data would provide an ever-renewed record for updating inspection methods, the value of "walk-around" inspections in specific locations and an "input" to the choice of another inspection method.

Figure 3.8 data could provide a baseline for reducing the uncertainty in the detection of large damage, and potentially for updating the probability of severity factors and zones.

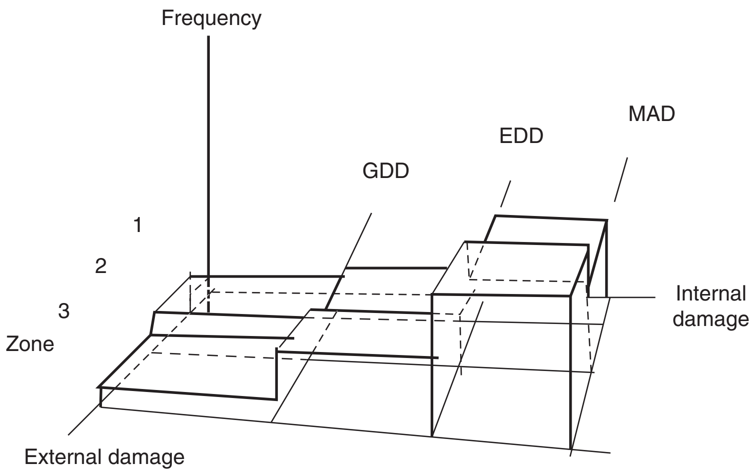

Figure 3.7. Damage data; external vs internal.

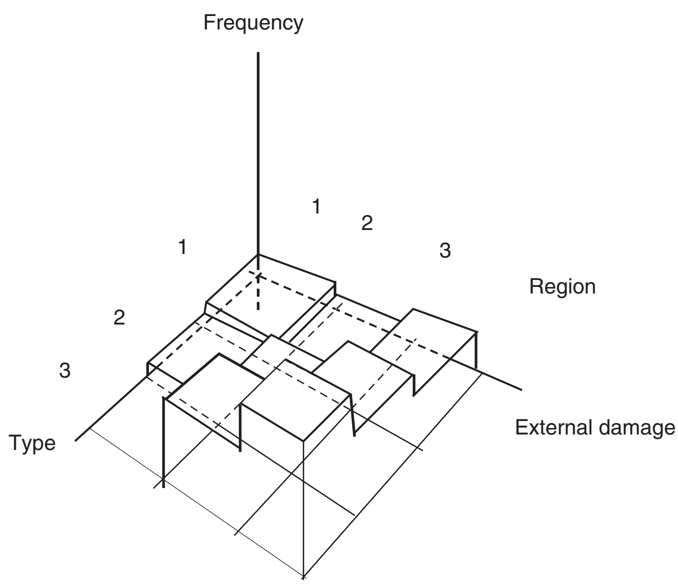

Figure 3.8. External damage.

Figure 3.9 can be interpreted as the effectiveness of an inspection method and of its adaptation to service environment in specific locations. Feller (1971, p. 177) shows how the plotted function can be interpreted as the distribution function of detected damage sizes. The data can therefore serve two purposes and especially provide guidance in updating the probability density for detection of local damage.

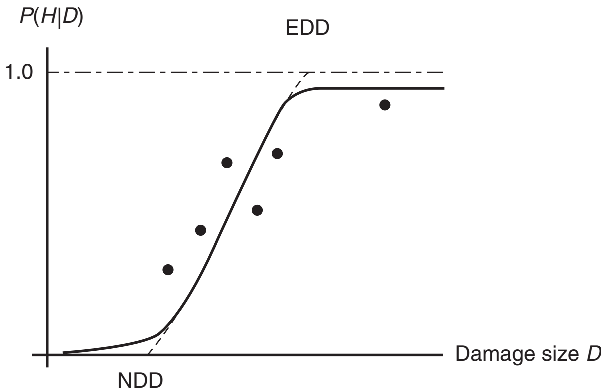

Figure 3.9. Detection vs damage size.

Service records mostly contain damage data, while residual strength is the interest focus. Stirzaker (2003) describes methods for using damage data to make inferences about residual strength, extending the usefulness of monitoring and updating of safety bases.

A well-planned monitoring, feedback and updating program can be an effective tool in the preserving of safety levels in service.

Chapter 4

Innovation

Structural design should be, and often is, a creative process for which empiricism has limited value. Innovation is the "life-blood" of better designs, but comes with considerable challenge to structural safety, because "the validating service experience" is not available.

One way to compensate for the lack of service experience is to introduce explicit safety constraints into the structural design process. To design so that a specified level of safety is reached and then maintained in service by a complementing risk management process. The risk management can be established as a control process that uses flexible inspection programs that, based on feedback from service, can be changed to method and frequency to keep probability of an unsafe flight within required levels.

4.1. SERVICE EXPERIENCE

Pertinent service experience provides a validation of design methods, design criteria, detail designs, structural integrity rules, definitions of loads, environmental requirements, manufacturing processes and criticality decisions. Consequently, the lack of service experience must be overcome. A prudent introduction, into the design process, of explicit safety constraints, "high-fidelity-analysis" methods and design data testing that is calibrated to these constraints could "go a long way" toward compensating for lack of service experience.

Emerging service experience and "new" data are important factors in maintaining safety levels, but also in the learning process that produces an improved future. A rational process monitors, collects, analyzes and produces feedback into risk management and design processes.

Another important aspect of service experience is the validation of "Fail-Safe design" principles. FAR and JAR both require fail-safety, if *B*-value "allowables" are to be used in the design. Demonstration of compliance with fail-safe requirements is a very complex and difficult undertaking. Success is very sensitive to material selection, processes, material mixtures and detail designs. The sheer nature of fail-safety precludes a "total compliance demonstration" by test. So, high-fidelity-analysis alternatives for damaged structure must be made available. Breakthroughs produced by NASA Langley Structural Mechanics Department (formerly lead by Dr J.H. Starnes, Jr) in the "high-fidelity-analysis" field, provide a natural avenue.

Scenario-based inductive design methods and detail design approaches for damage containment must also be part of the future of safe innovation. Lack of service experience is an integral part of innovation, and the methods to be used in design must be able to account for safety explicitly.

4.2. CRITICALITY

Modern aluminum structures have developed to the point where "quality" and validated detail designs have become the answer to fatigue, while sizing of "acreage structures" is based on ultimate strength. Composites, however require a more complex approach. A typical case for composites is shown in Figure 4.1. Figure 4.1 shows,

$$1.5 \cdot \text{LLR} \leq \text{ULC} \tag{4.1}$$

where ULC refers to ultimate load capability. Eq. (4.1) then is an illustration of a damage tolerance critical PSE, because sizing to ULC would produce less thickness than what damage tolerance requires. As this is a typical situation for composites, an optimized selection of the damage sizes MAD and MUD would require a selection of required probability levels of the curve in Figure 4.1. ULC is obviously defined by:

$$\Pr(s \leq \text{ULC}) = 0.10 \tag{4.2}$$

(where s represents strength), as *B*-values are required. The probability requirements for LLR must be derived from the requirement of a maximum level of the probability of an unsafe flight.

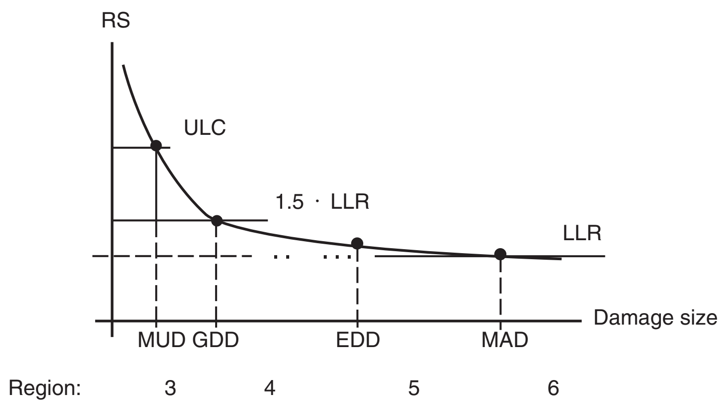

Figure 4.1. Damage size vs residual strength, RS.

The guided evolution of the 2000-series aluminum in "traditional, tension–critical structure" has resulted in a situation where,

$$1.5 \cdot \mathrm{LLR} \approx \mathrm{ULC}$$

based on the time-honored requirement for residual strength of a "panel" with one failed "central stringer" and a cracked skin-width equal to the stringer spacing. For composites, in both tension and compression critical structure, there is a lot of variety. An engineering process supporting a balanced selection of damage size intervals is a must for the efficient formulation of design criteria.

The overall safety requirements for the vehicles impose values for p in,

$$\Pr[\mathrm{RS} \leq \mathrm{LLR}|D_5] \leq p$$

where

$$D_5 \text{ is } \mathrm{EDD} < D_\mathrm{s} < \mathrm{MAD}$$

and could be considered the focus interval for residual strength integrity. Then,

$$D_4 \text{ is } \mathrm{GDD} < D_\mathrm{s} < \mathrm{EDD}$$

is the interval where the requirements of probabilities for growth rates are the most important. A detailed discussion of effects and orders of magnitude can be found in Chapter 2.

A damage tolerance criterion development resulting in high confidence levels would pay attention to the slope of,

$$\mathrm{RS} = \mathrm{RS}(d_\mathrm{s})$$

in the pertinent region. For regions where,

$$\mathrm{RS}'(d_\mathrm{s}) = 0$$

the potential for a very damage tolerant design is attractive, however the residual strength maybe too small, or detectability may fall short of desirable. So, again the need for a balanced selection of intervals has to be emphasized.

Another major concern is the shape of the function in regions 3 and 4. Region 3 could be based on the definition that emerges from:

$$\mathrm{ULC} \text{ being based on } F_B(1 + \mathrm{MS}) \text{ and LLR on } F_{\mathrm{lim}}(\text{limit allowable})$$

and Figure 4.1 is interpreted as an allowables capability, i.e. Pr(RS < LLR) = 0.10. If we return to Figure 4.1 and conclude that the maximum ultimate stress, if damage tolerance is critical, is $1.5 \cdot$ LLR and the ultimate margin of safety could be written,

$$\mathrm{MS} = \frac{F_B}{1.5\mathrm{LLR}} - 1 \tag{4.3}$$

The case where equal criticality is equivalent to zero margin, we would find that maximum ultimate damage, MUD would be,

$$\mathrm{MUD} = \mathrm{GDD}$$

which would result in a very damage tolerant design with good detectability without any extra weight penalty, as limit loads design damage tolerant structure.

As an example, one could then have the following allowables probabilities:

Region	$\Pr(s < \mathrm{ULC})$	$\Pr(s < \mathrm{LLR})$
3	10^{-1}	10^{-4}
4	–	10^{-3}
5	–	10^{-2}

if the overall safety objective was of the order of magnitude of "One unsafe flight in one hundred thousand flights."

Figure 4.2 illustrates the way the balance in choice becomes dependent on the inspection approach, and allowable end-load ($N_x = t \cdot \mathrm{RS}$), as it would be natural to, e.g. use the inequality,

$$P(\overline{B}_T|D_5) \cdot P(\overline{H}_T|D_5) \le p_b \tag{4.4}$$

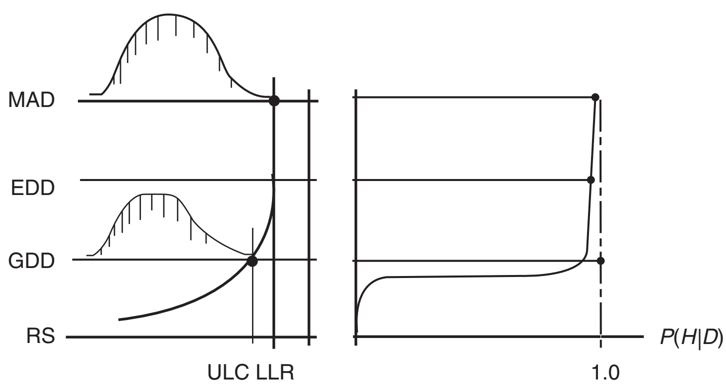

Figure 4.2. Residual strength and detection vs damage size.

4.3. DAMAGE TOLERANCE

Damage tolerance is often thought of as the capability of structure to carry loads with damage present. That makes accidental threats – the understanding of their nature and probability of inflicting damage – a large part of the quest for damage tolerance integrity, especially for region 5 (see Figure 4.1). Damage tolerance has several important interactions with damage resistance, damage containment and damage growth.

Innovation and the changing operating environment (both by location and in time) contributes to uncertainty of the requirements for damage tolerant designs. The guiding equation for probability can be expressed as,

$$P\left(\overline{B}_T\overline{X}_TD_{5T}\right) = P\left(\overline{B}_T|D_{5T}\overline{X}_T\right) \cdot P\left(D_{5T}|\overline{X}_T\right) \cdot P\left(\overline{X}_T\right) \tag{4.5}$$

The participating events are,

$\overline{B}_T$: RS $\leq$ LLR;
$\overline{X}_T$: Damage is present;
D_{5T}: The damage size is in region 5.

The first factor of the right-hand side of Eq. (4.4) can be looked at as a statement about the residual strength at a location of a PSE for a specific damage scenario, and could be given an allowables-like definition. It is apparent that, especially with new materials, new processes and/or structural concepts, that it would be "a big order" to produce all design data by testing.

An alternative that has considerable appeal is the approach that conducts the testing on the coupons and elements level and uses analytical "scale-up" to produce design data for PSE-size structures. Much progress has been made during the last decade in the areas of "high-fidelity-analyses" and "local/global" finite element work. Much of this work was lead by Dr J.H. Starnes, Jr at NASA Langley. It could be the basis for both scale-up in geometry and for support of scale-up of random behavior.

The second factor of the right-hand side of Eq. (4.4) depends on damage resistance and damage growth rates. It is,

$$P(D_{5T}|\overline{X}_T) \tag{4.6}$$

The last few years have seen a rise in types of emerging threats and increase in frequency of occurrence. The following examples should be considered in the design process:

- Construction debris on the runways;
- Tire fragments from the landing gear;

- Undetected impact by turbine fragments;
- Impact by large hailstones in flight;
- Unreported collisions with ground vehicles; etc.

A variety of different shapes must be considered in the evaluation of this probability and specifics should be part of design data and design criteria.

Eq. (4.4) can be rewritten as,

$$P(\overline{B}_T D_{5T} \overline{X}_T) = \sum_{i=1}^{n} P(\overline{B}_T D_{5T} \overline{X}_{iT}) \tag{4.7}$$

where the event,

$\overline{X}_{iT}$: Threat-dependent damage is present.

The threat-dependent version of Eq. (4.6) is,

$$P(D_{5T} | \overline{X}_{iT}) \tag{4.8}$$

and when external damage is present,

$$P(D_{5T} \overline{X}_{iT} \overline{E}_{iT} Z_{ijT}) \tag{4.9}$$

which is of interest in evaluating the seriousness of impact when external damage has been detected. Eq. (4.8) could be initialized and then monitored and updated as opportunities arise.

Finally, the third factor of every term of Eq. (4.7),

$$P(\overline{X}_{iT}) \tag{4.10}$$

can be considered to have some empirical support from previous service history, even in an environment of innovation.

An a priori value can be developed on that base, updated for "plausible" threats and validated in the exploratory design data testing.

Innovation provides many challenges in design, especially for damage tolerance critical safety levels. Plausible, initial design criteria must be developed for design and vigilant monitoring approaches must be in place to control safety levels and manage uncertainty.

4.4. INDUCTIVE METHODS

Chapter 1 showed the sufficient constraints for producing a safe design, and that includes safe operation. The safety discussion described how the safety constraints could be applied to structural integrity and expressed by,

$$\sum_{i=1}^{n} P\left(\overline{B}_T D_{5T} \overline{X}_{iT}\right) = \sum_{1}^{n} P\left(\overline{B}_T | D_{5T} \overline{X}_{iT}\right) \cdot P\left(D_{5T} | \overline{X}_{iT}\right) \cdot P\left(\overline{X}_{iT}\right) \tag{4.11}$$

which can be interpreted to represent threats and locations for a specific part of a PSE. Eq. (4.11) can be expressed in terms of constraints and written as,

$$\sum_{1}^{n} P\left(\overline{B}_T | D_{5T} \overline{X}_{iT}\right) \cdot P\left(D_{5T} | \overline{X}_{iT}\right) \cdot P\left(\overline{X}_{iT}\right) \leq p_j \tag{4.12}$$

where only region 5 is critical, if limits of the regions are selected wisely, and there is no degradation at work. If we assume that one threat and location can be identified as critical, we can write,

$$P\left(\overline{B}_T | D_{5T} \overline{X}_{iT}\right) \cdot P\left(D_{5T} | \overline{X}_{iT}\right) \cdot P\left(\overline{X}_{iT}\right) \leq p_{jc} \tag{4.13}$$

where for equal criticality of the different threats and locations, the constraint value becomes,

$$p_{jc} = \frac{p_j}{n} \tag{4.14}$$

We can see from Eq. (4.13) that the design methods must include probability statements about,

- Damage resistance;
- Damage growth rates; and
- Damage tolerance.

So it is possible to select material, process and a detail design approach that puts limits on the second and third factor of the left-hand side of Eq. (4.13). The next example will illustrate orders of magnitude.

Example 4.1: We select the equal criticality case, which gives,

$$P\left(\overline{B}_T | D_{5T} \overline{X}_{iT}\right) \cdot P\left(D_{5T} | \overline{X}_{iT}\right) \cdot P\left(\overline{X}_{iT}\right) \leq 10^{-5}$$

if we choose $n = 10$. Suppose in addition, that region 5 represents a rarely seen set of damage sizes, then we could say,

$$P(D_{5T}|X_{iT}) \approx 10^{-3}$$

and if the location is "exposed" we could estimate,

$$P(\overline{X}_{iT}) \approx 10^{-1}$$

and we get the requirement,

$$P(\overline{B}_T|D_{5T}\overline{X}_{iT}) \leq 10^{-1}$$

If we assume that the maximum applied end-load is $N_{\lim}$ we have,

$$N_{\lim} = \text{LLR} \cdot \bar{t}, \Rightarrow \bar{t} = \frac{N_{\lim}}{\text{LLR}}$$

where the result represents the total gage, which represents an allowable-like way to satisfy the safety constraints in design.

Residual strength versus damage sizes for limit load capability can be thought of as in the model scenario in Figure 4.3. A very important part of damage tolerance designs is a thorough analysis of possible scenarios. Figure 4.4 describes a segment of a wing PSE for a multi-spar construction.

Part of the design or analysis of the PSE involves determining either what is critical location for determining thicknesses or for a given thickness what is

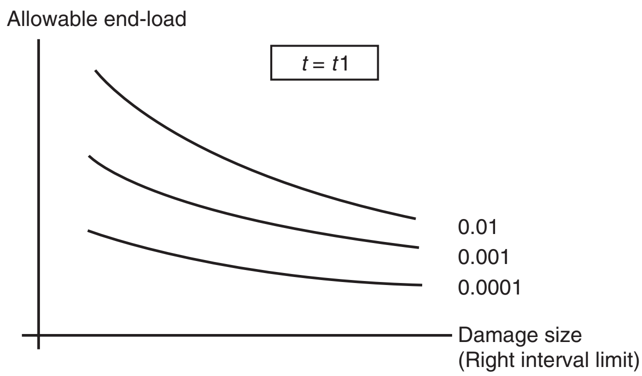

Figure 4.3. Allowable limit end-load for fixed thickness and interval.

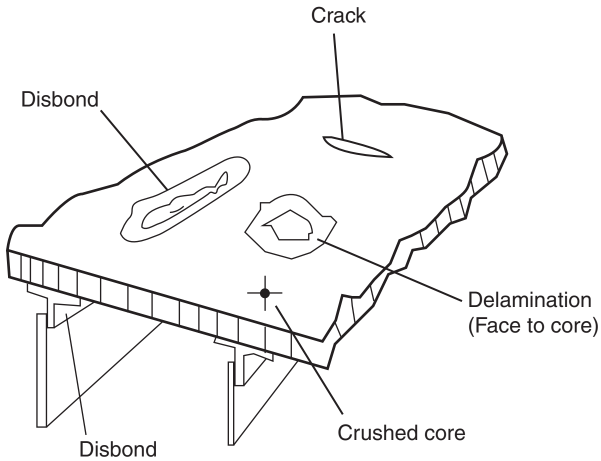

Figure 4.4. Damage scenarios.

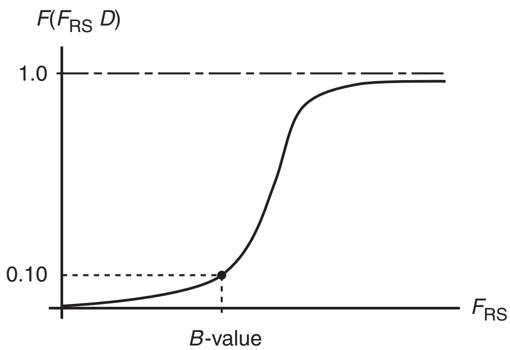

Figure 4.5. Residual strength allowable.

the probability of lost structural integrity or margin of safety for a given safety level,

$$\mathrm{MS} = \frac{F_{\mathrm{RS}}}{f_{\mathrm{lim}}} - 1$$

and the probabilities can be written as,

$$\Pr(\mathrm{RS} \leq \mathrm{LLR}|D_{5T}) \text{ or } \Pr(F_{\mathrm{RS}} \leq F_{\mathrm{lim}}|D_{5T})$$

Figure 4.5 shows an example of the distribution function for F_{RS}. The figure shows a *B*-value example.

Most of the traditional structural design uses allowables as a basis for sizing, and it also seems for damage tolerance composite design that their use will be less data demanding than classical inductive methodology, even though it requires a very close interaction between design criteria, damage scenarios, inspection methods, testing and ''scale-up.''

Chapter 5

Safety Objectives

The policy of innovation has often been expressed in terms of safety as:

"Composite Structure shall be as safe as or safer than the structure it replaces."

Objectives, like the one above, require an explicit measure of safety. Figure 5.1 illustrates the structural states that can be used as a basis for definitions of safe or unsafe structure.

The probability density has two "branches"; detected, D, or undetected, ND. The definition of an unsafe state is:

"An unsafe state is the state of undetected unacceptable integrity."

Structural integrity can be expressed in terms of residual strength, and Figure 5.2 contains a residual strength surface of a specific probability value.

5.1. SAFETY AS A FUNCTION OF TIME

Figure 5.2 shows the effect of time on residual strength. Figure 5.2 shows a limit load capability, LLC which should be the same as the limit load requirement, LLR. It also shows a growth curve that takes the PSE to unacceptable integrity by satisfying the inequality,

$$\mathrm{RS} \leq \mathrm{LLR}$$

If we now ask: "What is the probability of an unacceptable integrity for a PSE which has n potentially critical locations and 5 damage size regions?" The answer could look like,

$$P\left(\overline{U}_T\right) = \sum_{i=1}^{n} \sum_{j=1}^{5} P\left(\overline{B}_{iT} | D_{jT}\overline{X}_{iT}\right) \cdot \left(D_{jT}\overline{X}_{iT}\right)$$

Figure 5.2 shows that both damage growth and degradation makes residual strength and consequently safety functions of time.

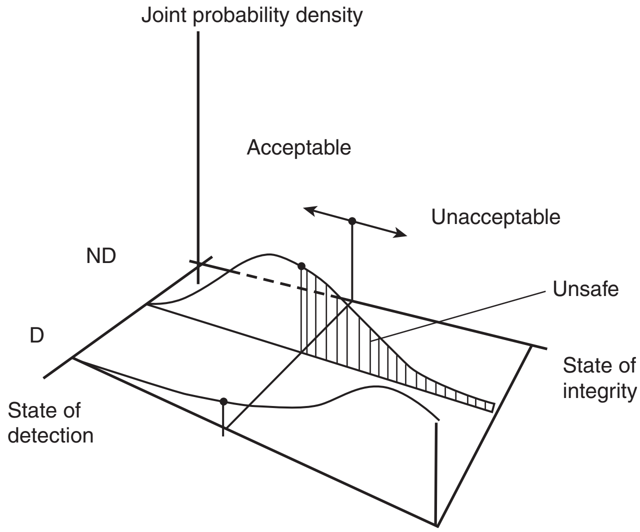

Figure 5.1. Unsafe state.

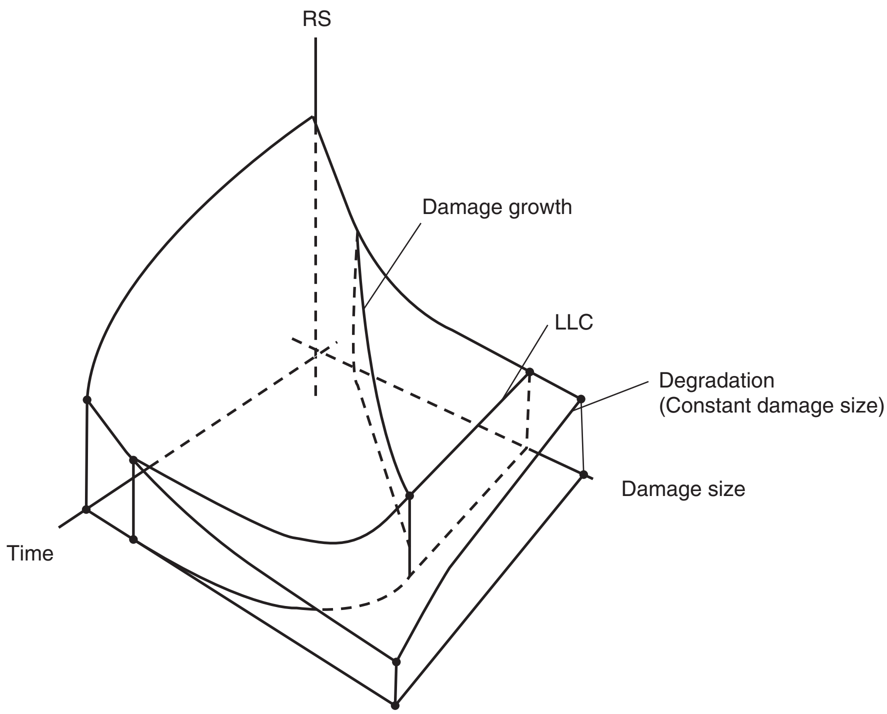

Figure 5.2. Probability surface for residual strength.

Figure 5.1 defines the Probability of "Unsafe Flight,"

$$P(\overline{S}_T) = P(\overline{U}_T \overline{H}_T)$$

which can be expanded to:

$$P(\overline{S}_T) = P(\overline{U}_T \overline{H}_T \overline{H}_\tau) \tag{5.1}$$

Eq. (5.1) is a measure of safety after a major inspection at T (τ is the time for the previous inspection). Between inspections, the probability of an unsafe flight increases, and time till next inspection can be used to control the maximum probability for an unsafe flight.

5.2. INSPECTION

Eq. (5.1) illustrates the importance to safety of major inspections. Practically, it allows us to detect and repair damage before structural integrity is being threatened or lost. It also makes it possible to set realistic safety objectives and design constraints.

The description of a specific inspection method must come with data like those described in Figure 5.3, and be specific to structural concept and type of damage. Where

NDD is Not Detectable Damage;
MUD is Maximum Ultimate Damage;

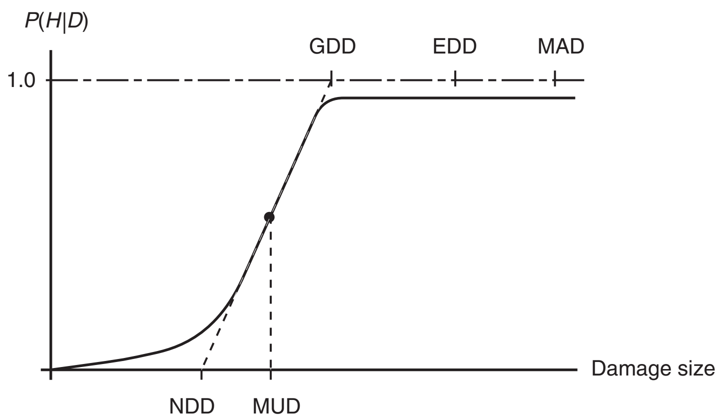

Figure 5.3. Definition of inspection method.

GDD is Good Damage Detectability;
EDD is Easily Detectable Damage;
MAD is Maximum Allowed Damage (for limit capability).

While Figure 5.3 is free from uncertainty in aluminum thin-gage structure, the "composite world" is more complex. The situation often is such that there is both external and internal damage. The relation between them depends on shape, size, consistency, direction and speed of the impacting object, and the traditional selection of a "one-inch-spherical-impactor" for determination of BVID is far from a realistic representation of service environment.

A practical engineering approach to design of safe, composite structure must differentiate between "fresh" accidental damage and damage sizes established by growth. Figure 5.3 then would be the representative of internal damage (damage "controlling" residual strength) while "fresh" accidental damage could be described by Figure 5.4.

Large accidental damage in composite structure requires special data to characterize inspection methods in support of safety. Large accidental damage inflicted in service often involves damage locations accessible to "walk-around" preflight inspections. The characterization of the walk-around inspection quality needs to be documented and implemented as described in Figure 5.5, recognizing that mostly only external damage "enters the picture."

The argument, used for the one-dimensional case, can be extended to the two-dimensional case. Figure 5.5 shows how the uncertainty in external damage

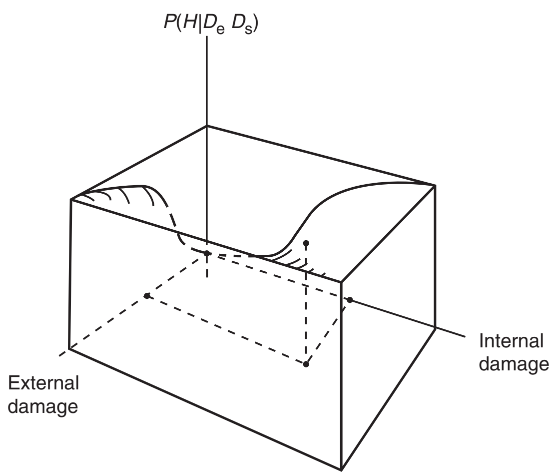

Figure 5.4. Probability of detection vs internal and external damage.

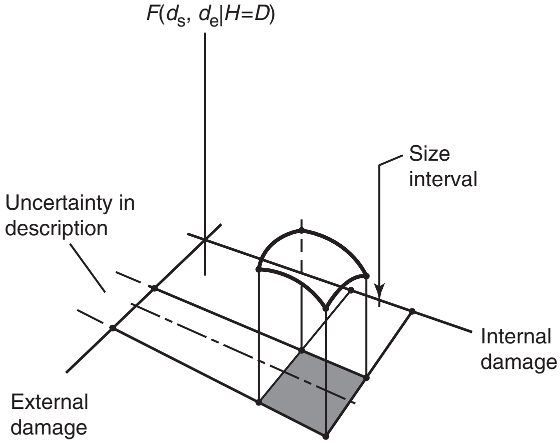

Figure 5.5. Distribution of damage.

characterization and a probability of detection requirement can be translated to an internal damage size interval that can be used for repair decisions–residual strength determination.

5.2.1. "Walk-around" inspection

The walk-around, preflight inspection is a very important part of safety, design and survival with damage. A closer study of the detail probabilities is warranted. They can be expressed in terms of:

"The probability of not failing on the kth flight, $P(\overline{F}_k)$."

It can be expressed in terms of detail events as,

$$P(\overline{F}_k) = \sum_{i=4}^{5} \sum_{j=1}^{3} \left[P(\overline{H}_k \overline{B}_k S_k S_{Dkij}) + P(H_k S_{Dkij}) \right] \tag{5.2}$$

where

$\overline{H}_k$: Not found on the walk-around inspection before the kth flight;
$\overline{B}_k$: RS $\leq$ LLR on the kth flight;
S_k: Survival of the kth flight;
S_{Dkij}: State ij of damage on flight k;
H_k: Damage found.

and,

$$S_{Dk} = \overline{X}_k[(D_{4k} \cup D_{5K})(D_{e1k} \cup D_{e2k} \cup D_{e3k})] \Rightarrow P(S_{Dk}) = \sum_{i=4}^{5}\sum_{i=1}^{3} P\left(\overline{X}_k D_{ik} D_{ejk}\right)$$

and,

$$P\left(\overline{H}_k \overline{B}_k S_k S_{Dkij}\right) = P\left(\overline{H}_k | S_{Dkij}\right) P\left(S_k | S_{Dkij} \overline{B}_k\right) P\left(\overline{B}_k | S_{Dkij}\right) P\left(S_{Dkij}\right),$$

$$\text{and } P(H_k S_{Dkij}) = P(H_k | S_{Dkij}) P(S_{Dkij}) \tag{5.3}$$

Eqs. (5.3) are based on the assumption that no degradation in strength takes place without the presence of damage. They describe the two events: "Not detecting damage but surviving flight k" and "Detecting the damage during the preflight inspection before flight k," which together assures safe completion of flight k.

Example 5.1: This example focuses on Eqs. (5.3) to demonstrate the effects of the two types of damage; external and internal. As this is a "walk-around inspection" we find for $j = 1$ that

$$P\left(\overline{H}_k | S_{Dki1}\right) = 1$$

And the nature of the inspection only deals with external damage sizes. An inexpensive technology involving some hand-held equipment would change the safety picture dramatically. We also find that,

$$P(H_k | S_{Dki1}) = 0$$

Pursuing the assessment with a consistent set of numerical values that would be compatible with the safety objective being developed as a baseline for orders of magnitude, studied in this book, results in the following starting point for walk-around inspections.

For

$$\begin{array}{lll}
i = 4 & j = 1 & 1 \cdot 0.9 \cdot 10^{-2} \cdot 10^{-2} + 0 = 0.9 \cdot 10^{-4} \\
 & j = 2 & 0.5 \cdot 0.9 \cdot 10^{-2} \cdot 10^{-2} + 0.5 \cdot 10^{-1} \\
 & j = 3 & 0.2 \cdot 0.9 \cdot 10^{-2} \cdot 10^{-2} + 0.3 \\
i = 5 & j = 1 & 1 \cdot 0.9 \cdot 10^{-1} \cdot 10^{-3} + 0 \\
 & j = 2 & 0.5 \cdot 0.9 \cdot 10^{-1} \cdot 10^{-3} + 0.5 \cdot 10^{-1} \\
 & j = 3 & 0.2 \cdot 0.9 \cdot 10^{-1} \cdot 10^{-3} + 0.5
\end{array}$$

So the probability of not failing during flight k becomes,

$$P(\overline{F}_k) \approx 0.9$$

As we see in this range detection dominates the value, and the outlook is reasonable if one can maintain the inspection quality on this level, which would yield a probability of surviving k flights with lost integrity as,

$$P(\overline{F}_{1k}) = 0.9^k, \text{which for 10 flights is 0.35}$$

The purpose of this example is to illustrate the importance to safety of "walk-around inspections," and to show the orders of magnitude for which to aim in the requirement definition.

The message of this section is the importance of considering the type of damage, the type of inspection possible and the definition of detection effectiveness when creating design criteria and making design decisions.

The uncertainty in translating laboratory information to useful data in service is also an important function of monitoring and uncertainty reduction.

5.3. ACCIDENTAL DAMAGE

In the traditional world of aluminum structures, accidental damage, beyond what is specified in the "discrete source damage" regulations in FAR and JAR, is a very minor part of damage tolerance design. In the world of composites, however, accidental damage plays a large role and comes in "many flavors":

- Manufacturing flaws;
- Transport damage;
- Damage during maintenance and repair;
- Damage in service,

and they can be classified as damage to be accounted for as part of:

- Ultimate integrity;
- Limit integrity;
- "Get-home" load integrity.

Reporting and detecting are less than perfect processes, both in manufacturing and service and they are very important factors both in design and service.

Therefore, accidental damage is a major threat to structural safety. The threat by an accidental damage involves,

- Damage tolerance;
- Fail-safety;
- Damage resistance;
- Damage growth,

and becomes a very important part of the design details and structural concepts. A way to include damage tolerance into the design process is by characterization in damage size regions and severity zones resulting in damage identification for:

- Ultimate load capability requirements;
- Limit load capability requirements;
- "Get-home" load requirements.

Fail-safety requirements could typically be satisfied by alternative load paths and redistribution provisions, thereby legitimizing the use of *B*-value allowables. Damage resistance would have to be assured by containing damage for specific threats within prescribed size intervals.

Finally, the damage growth rates for different environments must be contained within "reasonable" scatter intervals. Furthermore, the inspection programs must be designed so that internal damage can be routinely detected in the "limit regions," even if no external damage is present.

The most serious damage is the one that causes loss of integrity soon after a major inspection. If it happens in service, it is reasonable to assume that typically the damaged area would be accessible to "walk-around" inspections and detectable in a few flights. Safety would require closely guarded requirements on quality of walk-around inspections for that very reason.

That brings us to damage inflicted between inspection and first flight (during repair or regular maintenance). The natural defense against that type of damage would be quality assurance in maintenance, so that this event would be very rare.

The remaining damage causes involve growth of undetected damage. In these cases it would be moderate enough to assure several opportunities for detection during the period that probability of loss of integrity would remain moderate.

Design criteria must contain requirements for safety that includes all types of accidental damage and their effects on:

- Damage tolerance;
- Fail-safety;

- Damage resistance; and
- Damage growth,

in a way that assures Structural Integrity.

5.4. DESIGN DATA AND ALLOWABLES

An often used approach to Damage Tolerance of composites has been to have a "damaged-panel-test-program" and use the lowest value as a "cut-off" for sizing. However, as ultimate data must have a B-value quality. We will use the next example as a demonstration of the "goodness" of the approach.

Example 5.2: We assume that we have a normal distribution. We have decided to test n panels, and the results in order of value are:

$$y_1 \leq y_2 \leq \cdots \leq y_n$$

The probability density function is $f(x)$ and the distribution for y_1 is,

$$g(y_1) = n[1 - F(y_1)]^{n-1} \cdot f(y_1)$$

The probability that this process will lead to an allowable larger than the B-value is,

n	$\Pr(y_1 \geq F_B)$
5	0.59
10	0.35
20	0.12

showing that a "normal" allowables program would most likely be more effective, if a thoughtful planning and scale-up were to be used.

The objective behind the design data is to produce PSE-level data for the sizing part of the design. The assumption is that scale-up will be used and the next example illustrates how the basis for damaged element data could look.

Example 5.3: It is assumed that the vehicle requirements have produced a structural integrity requirement,

$$10^{-5} \geq P(\overline{U}_T) = P(\overline{B}_T | D_{5T}\overline{X}_T) \cdot P(D_{5T}|\overline{X}_T) \cdot P(\overline{X}_T) \tag{5.4}$$

where

$\overline{B}_T$ is the event RS < LLR;
D_{5T} is the region EDD < D_s < MAD, which represents the only critical interval at the critical location.

So the test data will be covering region 5 and wise selection of bounds now becomes apparent. The second factor on the right-hand side of Eq. (5.4), in this case represents a rare event,

$$P\left(D_{5T}|\overline{X}_T\right) \geq 10^{-3}$$

and if the location is exposed to accidental damage, then,

$$P\left(\overline{X}_T\right) \geq 10^{-1}$$

and the residual strength requirement would be,

$$P\left(\overline{B}_T|D_{5T}\overline{X}_T\right) \leq 10^{-1} \quad (5.5)$$

which would correspond to B-value. A choice of region made to allow for some amount of damage growth together with no degradation, would make the effect of time minimal.

Figure 5.6 describes design space, criticality and a comparison between ultimate and limit requirements from a design standpoint.

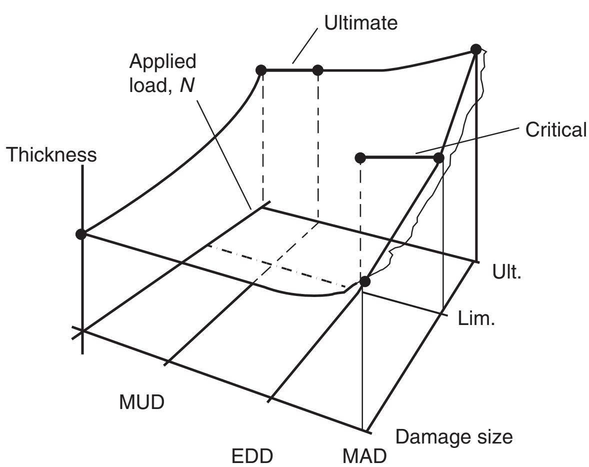

Figure 5.6. Design space and criticality.

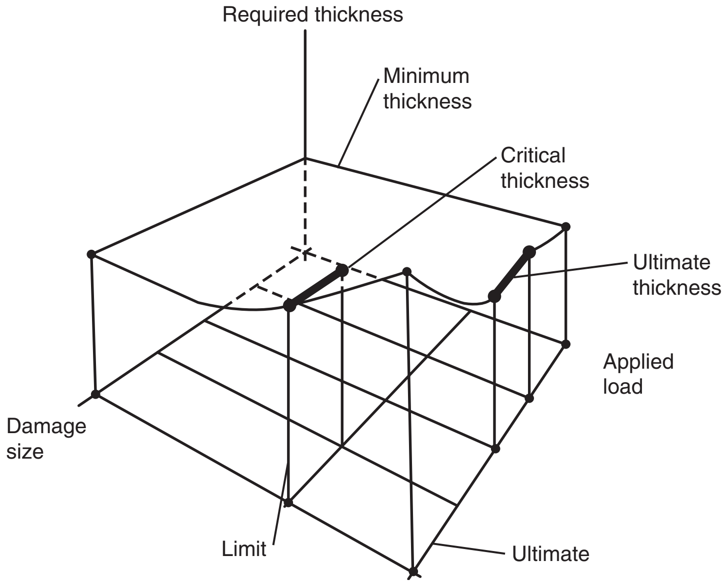

Figure 5.7. Regional design.

The surface represents a specific probability level and can be an implicit function of time, as damage changes in time. Figure 5.7 supports previous discussions on damage intervals. It also can be interpreted as an alternative to reduce some dependence on time. It illustrates the importance of selecting damage regions and ranges so that a realistic basis for the design and inspection criteria can be put in place.

Considering the advantages of designs using regions of damage sizes in the design criteria, it should be seriously considered, as data requirements for design information would be significantly reduced.

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Chapter 6
Risk Management

Risk management is a very important aspect of maintaining safety at an acceptable level in service. Figure 6.1 describes a control process that is based on changes in inspection approaches and periods.

The major inspections are selected by type and approach so that the lower bound value, LB, is not violated and the interval is selected so that the upper bound, UB, is not exceeded. So when service data are used to update knowledge and reduce uncertainty, inspection methods and periods can be used as the means of a control process to maintain safety in service.

6.1. UNSAFE STATE

The probability of an unsafe state (or unsafe flight) is the "measure of safety" that is the basis for the design to explicit safety constraints. Figure 6.1 illustrates the concept, and the probability of an unsafe state during the flight just before the inspection at time T is,

$$P\left(\overline{U}_T\overline{H}_t\right) = P\left(\overline{U}_T\overline{H}_t\overline{X}_t\right) + P\left(\overline{U}_T\overline{H}_tX_t\right) + \Delta \tag{6.1}$$

where the first term of the right-hand side of Eq. (6.1) describes the effect of damage, and can be written as,

$$\sum_{j=3}^{5} P\left(\overline{U}_T|\overline{H}_t\overline{X}_{tj}\right) \cdot P\left(\overline{H}_t|\overline{X}_{tj}\right) \cdot P\left(\overline{X}_{tj}\right) \tag{6.2}$$

where j represents damage size regions 3, 4, 5.

The second term describes "degradation" in properties without measurable mechanical damage sizes. The third term deals with accidental damage during operation, and can be written as,

$$\Delta = \sum_{k=1}^{n} P\left(\overline{U}_T Y_k D_{5kT}\overline{H}_{kT}|X_t\right) \cdot P(X_t) \tag{6.3}$$

The letter k refers to the kth flight after the last major inspection. The following sub-events are involved,

U_T: Acceptable integrity at T;
Y_k: Accidental damage inflicted on flight k;

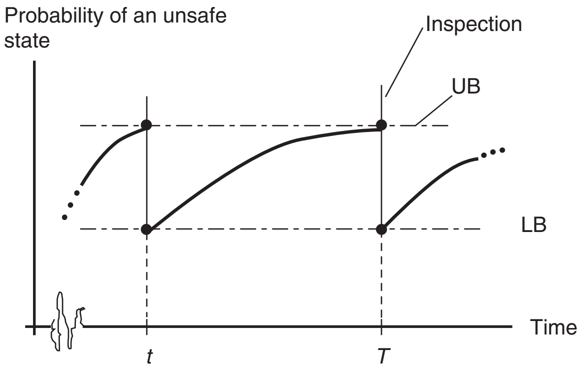

Figure 6.1. Probability of an unsafe state.

D_{5kT}: Damage size grows into region 5 by time T;
H_{kT}: Damage detected between k and T;
X_t: No damage present at time t.

An arbitrary typical term in the sum is,

$$P\left(\overline{U}_T|Y_kD_{5kT}\overline{H}_{kT}X_t\right)\cdot P\left(\overline{H}_{kT}|D_{5kT}Y_kX_t\right)\cdot P(D_{5kT}|Y_kX_t)\cdot P(Y_k|X_t)$$

Example 6.1: Suppose that, the location is not available for "walk-around" inspections and that accidental damage is contained to region 3, then we find that $\Delta = 0$, and the total effect is due to Eq. (6.2), which when using the same orders of magnitude as in Chapter 2 produces a value of $3 \cdot 10^{-9}$.

However, if the probability of not detecting an accidental, random damage with a damage size in region 4 is 0.5, then the probability of missing it in ten consecutive flights is less than 10^{-3}, and the contribution from Eq. (6.3) is,

$$\Delta \approx 10^{-12}$$

The keen-eyed designer, at this time, asks; "What about the accidental damage in maintenance or repair, between a major inspection and the first flight after it?"

This is one of the important decisions in the extended design process. The choices could be between quality control during this activity such that the probability of this type of damage being present at the first flight, is small. Or it could be "designed-in" damage resistance that would reduce the probability of inflicting threatening damage. A third alternative deals with constraining damage growth rates, either by detail protection or by reducing stresses.

The objective of the design is to reduce the probability of an unsafe flight due to structural design inadequacies, and the tools available are:

Damage tolerance increases;
Damage resistance improvements;
Damage growth rate reductions;
Quality control improvements;
Operating environment with safe protection;
Improved detection methods.

6.2. ROLE OF INSPECTIONS

Figure 6.1 illustrates the importance of major inspections, approaches and periods, and describes their role in maintaining safety. Example 6.1 alludes to the effects and value of "walk-around" preflight inspections. It even numerically shows that preflight inspections of modest quality can have large impact on the safety level retention between major inspections.

Figure 6.2 describes the important features of an inspection method and how they interact with the structural design. The emphasis is on composite structure, for which we attempt to select "key" damage sizes so that:

1. Maximum Ultimate Damage size, MUD, is selected so that ultimate and damage tolerance have equal criticality and that the probability of detection is "reasonably" large;

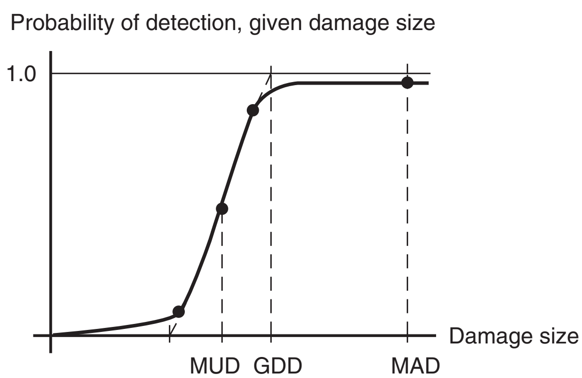

Figure 6.2. Inspection method characteristics.

2. Maximum Allowed Damage size, MAD, so that the damage resistance levels contain the damage, by "known" threats, to sizes below MAD;
3. The interval between Easily Detectable Damage, EDD, and MAD, contains damage probabilities compatible with the required safety level, at the same time, as the associated residual strength is insensitive to minor changes in damage size.

If we now assume that "walk-around" inspection is not possible at the PSE in question, we can express the probability of an unsafe state at t (between two major inspections) as the following sum of effects.

An unsafe state at time t, is,

$$P(\overline{S}_t) = P(\overline{U}_T\overline{H}_T\overline{X}_T) + P(\overline{U}_t U_T \overline{H}_T X_T) + P(\overline{U}_t | U_T\overline{H}_T\overline{X}_T) \cdot P(U_T|\overline{H}_T\overline{X}_T) \cdot P(\overline{H}_T|\overline{X}_T) \cdot P(\overline{X}_T) \tag{6.4}$$

where the first term represents the lower bound in the control process. The second describes accidental damage between inspection at time, T and the time t, and loss of integrity. The third involves growth of some undetected damage between inspection and loss of integrity. As we can see from Eq. (6.4) "non-detection" enters into every term and almost every factor either implicitly or explicitly.

So, clearly any respectable intent to establish and control "Level of Safety" involves thorough knowledge of and control of the inspection approach.

6.3. FUNCTION OF TIME AND INSPECTION APPROACH

Eq. (6.4) is a function of time. The second term on the right-hand side depends on time and is described in Figure 6.3. It represents behavior along the $d_s = 0$ line (no damage is present) as an extension of the LLR curve would intersect the time axis.

The third term (the first factor) is also a function of time, described in Figure 6.3 by the "Damage Growth" curve, the first factor deals with "loss of structural integrity," which can be derived from "acceptable structural integrity," (keeping in mind the conditional probabilities in Chapter 2),

$$P(U_t) = P(B_t|D_t\overline{X}_t) \cdot P(D_t|\overline{X}_t)P(\overline{X}_t) + P(B_t|D_tX_t) \cdot P(X_t) \tag{6.5}$$

where the sub-events are,

B_t: RS > LLR;
D_t: $D_s <$ MAD;
X_t: No damage is present.

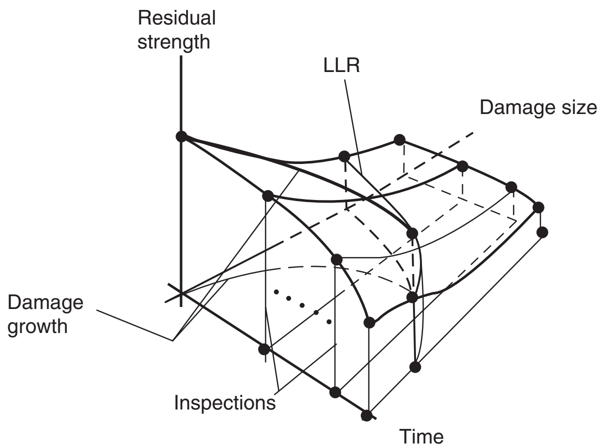

Figure 6.3. Residual strength as a function of time.

The probability of unacceptable structural integrity can be written as,

$$P(\overline{U}_t) = P(\overline{B}_t|D_t\overline{X}_t) + P(\overline{D}_t|\overline{X}_t) + P(\overline{B}_t|D_tX_t) \tag{6.6}$$

where the first term describes the probability of violating the LLR, given a damage size less than the "maximum allowed damage." The second term represents an excessive damage size for which the probability limit integrity is unacceptably small and the risk of "collateral damage" is large. Finally the third term covers the loss of LLC due to property degradation.

In the general case, all three terms would increase with time. If we now assume that t represents the flights between T and T_1, and that the PSE in question would not be available to "walk-around" inspection, then, the situation in Figure 6.4 would

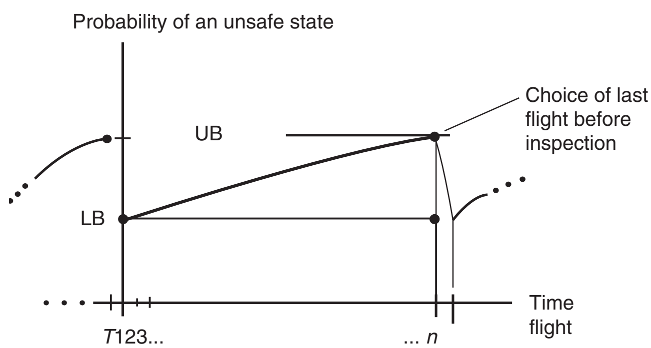

Figure 6.4. Choice of inspection interval.

be typical, and would involve the largest interval within the bounds of the control process.

6.4. UNCERTAINTY

Uncertainty affects risk management. Uncertainty originates with the need to make decisions with less than perfect data, especially in an environment of innovation. A situation of uncertainty is often unavoidable, as a quest for "Perfection" would make any engineering endeavor unmanageable, and the only alternative is making risk management a part of the "cradle to grave" engineering process in a disciplined way that not only provides focus but awareness of emerging safety threats. NASA has dedicated much effort toward this objective on the system level, and disciplines, like structures, now must follow in order to truly produce safe innovation.

Structural Design is a discipline that combines Philosophy, Science and Engineering, and the approach toward the future must use all three fields in a combined focus on reducing uncertainty in a way that serves safety.

Uncertainty comes in many "shapes." The following uncertainties are especially important to the design of composite structures,

- The part of total vehicle safety attributable to composite structure;
- The total need of design data testing due to the introduction of "new" materials, processes and/or structural concepts;
- The extent to which analytical "scale-up" methods can be used to produce PSE level design data and how much baseline testing should be done;
- The plausible accidental damage threats that should be included in damage tolerance and effective ways to produce damage resistant structure for realistic threats;
- The required validation of "High-Fidelity-Analyses" as substitute for service experience;
- The environmental long-term effect on damage growth rates;
- The selections of a priori probability distributions for residual strength and damage.

Some type of monitoring of service data (including inspection results) must be in effect and the feedback primarily must be used to reduce uncertainty and to improve understanding, short-term. However, there also must be educational objectives targeting the future state of knowledge serving the interests and safety of the general public. So, proprietary interests must yield to the common good. For example, the first entry of the list mentioned earlier can only come from the total picture of aviation, and will take a long time to finalize. It must be derived from many sources. The procedures for determining starting points, and the approach by which values are updated must be part of the FAA regulations.

The second entry can only be resolved long-term, also. The number of "surprises," in service for a specific model to the end of its service-life, is the true measure of performance. The safety objectives can be determined in a "conservative" way, and updated through a process that is part of international regulations.

The third entry involves initial demonstration of scientific validity of the specific application. However, the accumulation of service data identifying the unforeseen effects can take a long time. The short-term correction, if a safety issue is involved, can be handled through the risk management process. However, the long-term influence on general validity must be included in the "regulations" in terms of procedures required for demonstration of compliance.

The fourth and fifth entries cover "new" types of damage that constitute a threat to safety and presently not included in the regulations, e.g. damage caused by flight through a hailstorm. Introduction of specific threats in the regulations is the only safe way to assure long-term safety. Short-term effects on damage tolerance and resistance could be accounted for in the risk management approach. Validation and compliance demonstrations procedures belong in the international rules and guidelines.

The last three entries of the list are the ones that can acquire increasing certainty from monitoring and analyzing damage data (including inferences about growth). Damage data are the main source of feedback during service. Stirzaker (2003) shows an example of a method for updating distributions with new data which allows updates of the risk management and giving new inputs to the control process. And Rao (1973) describes methods on how damage data can be used to make inferences about residual strength.

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Chapter 7
Trades

Design of safe damage tolerance critical composite structure involves the process of trading different features to produce a balanced result. For example we could be faced with a situation that requires consideration of, damage tolerance, damage resistance, controlled damage growth and detectability. Figure 7.1 presents a partial picture of the design challenge which includes basing the resulting structural properties on a selected inspection method, and Figure 7.2 shows the conditional distributions for residual strength for one choice of regions.

Figure 7.1 is predicated on the use of damage regions 3 to 6 and the use of a minimum residual strength (allowables-like approach). The approach is described in Figure 7.2.

The situations after the inspection at τ are:

1. The structure is damaged, the damage is not detected and the structural integrity is lost, $\overline{X}_\tau \overline{H}_\tau \overline{U}_\tau$; *Reduce probability of occurring*;
2. The structure is damaged, the damage is not detected and the structural integrity is preserved, $\overline{X}_\tau \overline{H}_\tau U_\tau$; *Focus on the event of lost integrity at next inspection,* $\overline{X}_\tau \overline{H}_\tau U_\tau \overline{U}_T$; *Control safety by controlling damage growth rates in design*;
3. The structure is damaged, the damage is found, the structure is repaired, $\overline{X}_\tau H_\tau R_\tau$; *Focus on impact before next inspection,* $\overline{X}_\tau H_\tau R_\tau Y_k \overline{U}_T$; *Control safety by controlling damage resistance and damage growth through design*;
4. The structure is not damaged and the structural integrity is lost (degradation) $X_\tau \overline{U}_\tau$; *Make event improbable by designing "Safe Life"*;
5. The structure is not damaged and integrity is preserved, $X_\tau U_\tau$; *Focus on impact before next inspection*; *Control safety by controlling damage resistance and damage growth through design.*
6. The structure is damaged, the damage is not detected and the structural integrity is preserved or the damage is detected and repaired, or at least "tagged" for special attention.

This set of situations forms the baseline for design trades and we will investigate some of the possibilities.

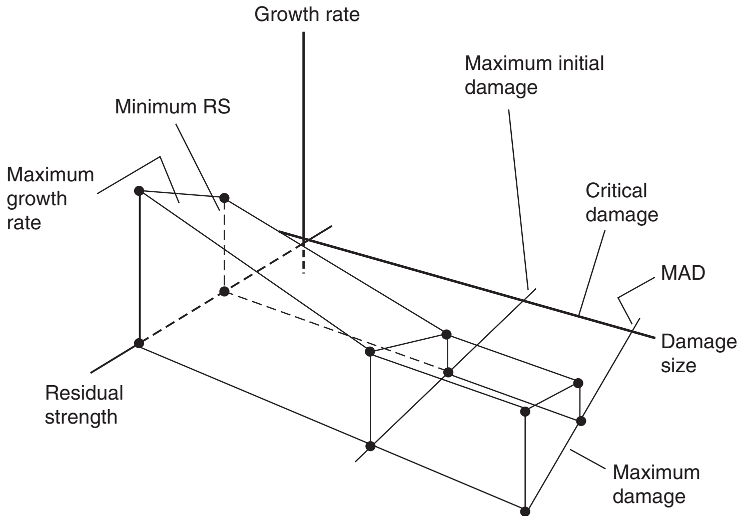


Figure 7.1. Design space.

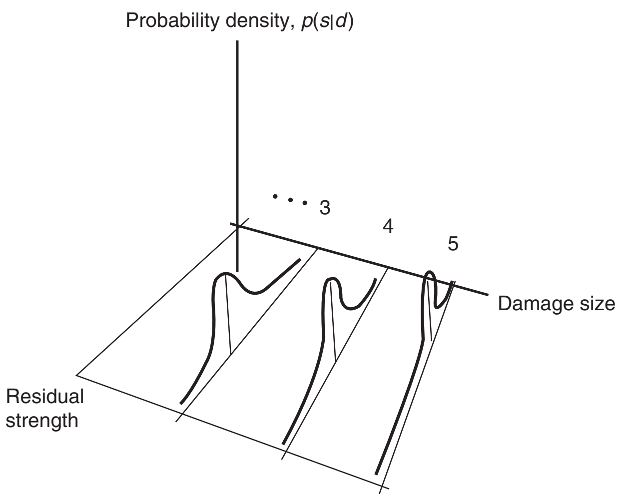


Figure 7.2. Residual strength distributions.

7.1. IMPACT

An accidental damage occurring between major inspections will result in one of the three different situations. The damage could be detected, the PSE could lose integrity or damage may grow, but integrity is preserved and damage is not detected at T.

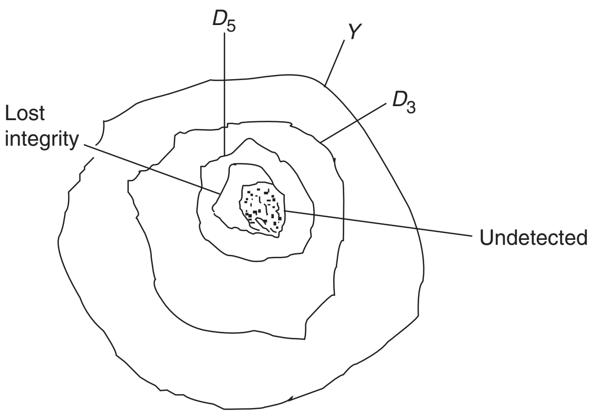


Figure 7.3. Set relations for sub-events in Eq. (7.2).

We will focus on the probability of an impact at k and an undiscovered loss of integrity at T,

$$P\left(Y_k\overline{U}_T\overline{H}_T\right) = \sum_{j\geq i}^{5}\sum_{i=3}^{4} P\left(Y_k D_{jT} D_{ik}\overline{U}_T\overline{H}_T\right) \tag{7.1}$$

where a typical term can be expanded as,

$$P\left(\overline{H}_T|D_{jT}\right)\cdot P\left(\overline{U}_T|D_{jT}\right)\cdot P\left(D_{jT}|Y_k D_{ik}\right)\cdot P(D_{ik}|Y_k)\cdot P(Y_k) \tag{7.2}$$

The first factor is the probability of not detecting the damage at T. The second factor is the probability of losing integrity given that the damage size is in region j. The third factor is the probability of growth from region i to region j between flight k and the inspection at time T. The fourth factor is the probability that initial damage size due to an accidental event at k will be in region i. Finally, the probability of an accidental event is $P(Y_k)$.

So, in order of Eq. (7.2), detectability, damage tolerance, damage growth, damage resistance and specific damage hazard enter into the design process. Example 7.1 will illustrate the sets involved in the "Trade" (see Figure 7.3).

Example 7.1: The number of situations can be reduced if we postulate that

$$P\left(\overline{U}_T|D_{4T}\right) \ll P\left(\overline{U}_T|D_{5T}\right)$$

$$P(D_{4k}|Y_k) \gg P(D_{3k}|Y_k)$$

$$P(D_{5T}|Y_k D_{3k}) \ll P(D_{5T}|Y_k D_{4k})$$

and left are $j = 5$ and $i = 4$. Going back to the previous examples for display of orders of magnitude we have,

$$P\left(Y_k \overline{U}_T \overline{H}_T\right) = 10^{-3} \cdot 10^{-4} \cdot 10^{-2} \cdot 10^{-1} \cdot 10^{-2} = 10^{-12}$$

which for an inspection interval of "3000 flights" yields the incremental probability of an unsafe state at T,

$$P\left(\overline{S}_T\right) = 3 \cdot c \cdot 10^{-9}, \quad \text{where} \quad 0 \le c \le 1$$

so if a selection of an inspection interval of 3000 is preferable, and the inspection method is set, then the residual strength requirement of LLR, the growth rate from 4 to 5 and initial damage in 3 together must be held below,

$$10^{-4} \cdot 10^{-2} \cdot 10^{-1} = 10^{-7}$$

so the product Pr(RS < LLR, D_5) · Pr(Growth from 4 to 5) · Pr(initial damage is in region 4) is for this case the basis for trades.

The order of magnitude of the growth rate is very important in establishing "allowables," and becomes an important influence and a troubling source of uncertainty.

Figure 7.4 describes one situation where maximum growth is from region 4 totally to region 5 in three inspection periods, and distributed uniformly. This assumption leads to the following probability of an accidental damage at k in region 4 to grow to region 5 before the next major inspection,

$$p_k = \frac{1}{4} \cdot \left[2^{[1-(k/n)]/[3-(1/n)]} - 1\right] \tag{7.3}$$

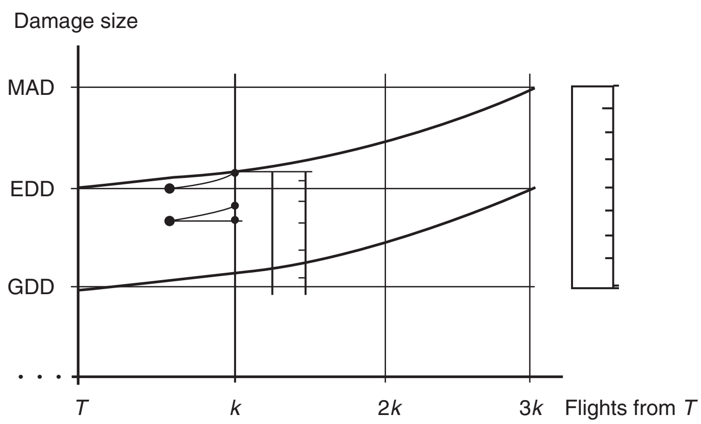


Figure 7.4. Growth from region 4 to 5.

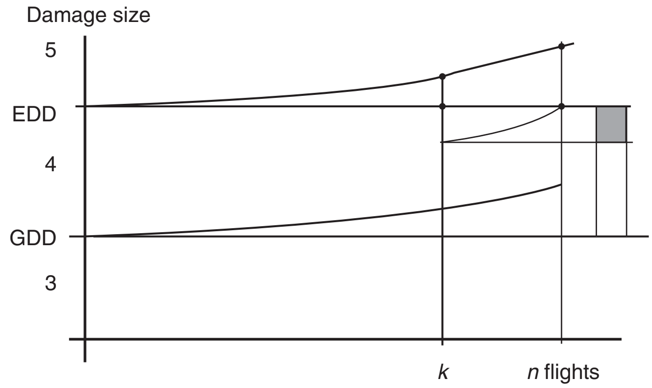


Figure 7.5. Interval for growth to region 5.

Figure 7.5 describes a situation for which damage sizes of growing damage is uniformly distributed between maximum growth rate and "zero-growth." The growth rate is assumed to be exponential and controlled during three inspection intervals. The focus is on the growth from region 4 into region 5 assuming that the probability for having entered region 5 from region 4 in three intervals is 0.5.

Eq. (7.3) yields the following values for the probability of growth to region 5 in k,

k	p_k
1	0.06
2	0.06
.	.
$0.8n$	0.012
$0.9n$	0.006
n	0

For this case we can conclude that $c \approx 0.9$, (in the expression of probability of an unsafe state at time, T given earlier). This reinforces how important it is to select realistic upper bounds for the "control process."

If we now look closer at "lost integrity," and write it as,

$$P(\overline{U}_T) = P(\overline{U}_T\overline{X}_T D_{5T}) + P(\overline{U}_T\overline{X}_T D_{4T}) + \cdots + P(\overline{U}_T X_T) \tag{7.4}$$

Suppose that,

$$P(\overline{U}_T\overline{X}_T D_{4T}) \ll P(\overline{U}_T\overline{X}_T D_{5T})$$

and that there is no degradation, then,

$$P(\overline{U}_T X_T) = 0$$

and we can write,

$$P(\overline{U}_T) = P(\overline{U}_T|\overline{X}_T D_{5T}) \cdot P(\overline{X}_T D_{5T}) \tag{7.5}$$

From the above we have,

$$10^{-4} \geq P(\overline{U}_T) = P(\overline{U}_T|\overline{X}_T D_{5T}) \cdot P(\overline{X}_T D_{5T}) = P(\overline{U}_T|\overline{X}_T D_{5T}) \cdot 10^{-3}$$

and,

$$P(\overline{U}_T|\overline{X}_T D_{5T}) = P(\overline{B}_T|\overline{X}_T D_{5T}) \leq 10^{-1} \tag{7.6}$$

so, if the choice is to use *B*-value allowables, and Eq. (7.5) was written for impact, especially, we would have,

$$P(\overline{U}_T) = P(\overline{U}_T|D_{5T}) \cdot P(D_{5T}|Y_k D_{4k}) \cdot P(Y_k D_{4k})$$

The growth (second factor), damage resistance (third factor) and the growth and damage resistance must be controlled to sustain integrity.

On the other hand, if you accept whatever growth and damage resistance the structure ends up having, then the residual strength allowable will have to be considerably smaller than the *B*-value.

7.2. DEGRADATION

Degradation refers to the process of reduction of mechanical properties by mechanical growth, like progressive micro-cracking, physical processes, like creep or by chemical processes like oxidation, breaking of bonds or forming of new bonds, all potentially deleterious effects. When these reductions happen without any detectable damage, the rules for "Safe Life" will take effect.

Among other things, this means that the time from the first flight till the loss of ultimate strength must have a safety factor (3 or larger, according to FAR 25). Example 7.2 illustrates one such situation.

Example 7.2: Suppose we are dealing with normally distributed variables, that a safety factor of 3 applies and the strength average reduces by 20 per cent in a three lifetimes, and that the standard deviation increases by 20 per cent in three lifetimes. What is the design value to use?

The mean after three lifetimes is 0.8μ, and the standard deviation is 1.2σ, then for a B-value the following is true,

$$\Phi(t) = 0.1 \to t = -1.3$$

And the design value should be based on

$$\frac{s - 0.8\mu}{1.2\sigma} = -1.3 \Rightarrow \frac{s}{\mu} = 0.8 - 1.2 \cdot 1.3 \cdot 0.10 = 0.64 \quad \text{if } C_v = 0.1$$

where a linear degradation is assumed. A remaining 64 per cent of the pristine allowable is a significant penalty. A few variations of degradation and design allowables, F_D, are shown in the table (C_1 is loss in the mean value and C_2 is the increase in the standard variation, both in three lifetimes).

Degradation		Coefficient of variation		F_D for $C_v = \ldots$	
C_1	C_2	0.05	0.10	0.05	0.10
0.2	0.2	•	•	0.72μ	0.64μ
0.2	0.1	•	•	0.73μ	0.65μ
0.3	0.3	–	•	–	0.53μ
0.4	0.4	–	•	–	0.42μ

It is clear, this design approach is far from efficient, and that a choice of protective treatment that reduces or eliminates the deleterious effects or a material choice that limits or eliminates degradation could be a better choice. This situation, maybe, requires a combination of surface treatment and new inspection technology, to be "practical."

7.3. DAMAGE UNDETECTED AT MAJOR INSPECTIONS

The principal design constraint (the lower bound) that minimizes the probability of "having an undetected loss of integrity," brings us to,

$$P\left(U_T \overline{X}_T \overline{H}_T \overline{U}_n\right) = P\left(U_T \overline{X}_T D_{5T} \overline{H}_T \overline{U}_n\right) + P\left(U_T \overline{X}_T D_{4T} \overline{H}_T \overline{U}_n\right) + \cdots$$

which represents the situation: damage is present, integrity is acceptable, damage is not detected at T and integrity is lost by the next inspection.

The first term on the right-hand side can be expanded as,

$$P(\overline{U}_n|U_T\overline{X}_TD_{5T}\overline{H}_T)\cdot P(\overline{H}_T|\overline{X}_TD_{5T}U_n)\cdot P(U_n|\overline{X}_TD_{5T})\cdot P(D_{5T}|\overline{X}_T)\cdot P(\overline{X}_T)$$
$$\approx P(\overline{U}_n|D_{5n})\cdot P(\overline{H}_T|D_{5T})\cdot 1\cdot P(D_{5T}|\overline{X}_T)\cdot P(\overline{X}_T) \qquad (7.7)$$

The second term can be expressed as,

$$P(U_T\overline{X}_TD_{4T}\overline{H}_TD_{5n}\overline{U}_n) = P(\overline{U}_n|D_{5n})\cdot P(U_T|D_{4T})\cdot P(\overline{H}_T|D_{4T})$$
$$\cdot P(D_{5n}|\overline{X}_TD_{4T})\cdot P(D_{4T}|\overline{X}_T)\cdot P(\overline{X}_T) \qquad (7.8)$$

The rest of the terms will be small as, $P(\overline{U}_n|D_{jn}) \ll P(\overline{U}_n|D_{5n})$ when $j<5$.

Example 7.3 will investigate a practical range of orders of magnitude.

Example 7.3: The major effects for this scenario are contained in Eqs. (7.7) and (7.8). Previously indicated orders of magnitude have the following results for Eq. (7.7),

$$10^{-3}\cdot 10^{-3}\cdot 10^{-3}\cdot 10^{-2} = 10^{-11}$$

Eq. (7.8) yields,

$$10^{-3}\cdot 1\cdot 10^{-2}\cdot 10^{-2}\cdot 10^{-1}\cdot 10^{-1} = 10^{-9}$$

where detectability and growth have the major inputs to a balanced design, while structural integrity derived from allowables-like consideration does not provide much room for trading.

The major "players" in this trade are damage resistance, damage growth rates and damage detection. This example illustrates the need for an overall strategy in the "target setting" for balanced designs and re-emphasize that the design "drivers" are: Damage tolerance, Damage resistance, Damage growth rates, Damage detection, Inspection method and Inspection period!

Example 7.4: Example 2.2 showed an illustration of establishing one desirable probability level. The example yielded $5\cdot 10^{-9}$ and we will base this example of that value.

Before setting the lower bound, we will investigate the increase between two major inspections. We will consider "a damage in region 4 at T and its growth" together with "accidental damage, initially in 4," during the period.

Example 7.1 showed how the accidental damage during the period dominated that case, and for an inspection period of 3000 flights the increment was $3 \cdot c \cdot 10^{-9}$; an approximation of the value is $2.5 \cdot 10^{-9}$.

So if the design objective is to stay under $5 \cdot 10^{-9}$, we have a lower bound of $2.5 \cdot 10^{-9}$. The result is:

$$P\left(\overline{B}_T | D_{5T}\right) \leq 10^{-1}$$

requiring a B-value residual strength allowable for region 5, if

- Damage resistance design have placed initial damage size in region 4 with highest probability;
- The inspection method is such that damage sizes in region 5 will be missed with a probability of 10^{-3};
- The inspection period is 3000 flights; The damage growth rates are maintained below what is shown in Figure 7.4.

This special case shows that a productive trade may be initiated between inspection quality and residual strength data quality.

7.4. REPAIR

If the repair policy in support of safety were: "If detected, repair!" we could use the following to describe the situation, $T_\tau = \overline{X}_\tau H_\tau R_\tau$, and focus could be on,

$$\begin{aligned} P\left(\overline{X}_\tau H_\tau R_\tau Y_k D_{4k} D_{5T} \overline{U}_T\right) = {} & P\left(\overline{U}_T | D_{5T}\right) \cdot P(D_{5T} | Y_k D_{4k}) \\ & \cdot P(D_{4k} | Y_k) \cdot P(Y_k) \cdot P(T_\tau) \end{aligned} \tag{7.9}$$

where T_τ is the total situation at first flight after the activities at τ, and can be expanded as:

$$P(T_\tau) = P\left(R_\tau | H_\tau \overline{X}_\tau\right) \cdot P\left(H_\tau | \overline{X}_\tau\right) \cdot P\left(\overline{X}_\tau\right) \tag{7.10}$$

If however, we wanted a more detailed repair policy we could focus on:

$$P(T_\tau D_w) = P(T_\tau D_l) + P(T_\tau D_3) + P(T_\tau D_4) + P(T_\tau D_5) + P(T_\tau D_6) \tag{7.11}$$

where an arbitrary term $P(T_\tau D_i)$ can be expanded as,

$$P(T_\tau) = P(T_\tau D_i) = P\big(R_\tau|D_i H_\tau \overline{X}_\tau\big) \cdot P\big(H_\tau|D_i \overline{X}_\tau\big) \cdot P\big(D_i|\overline{X}_\tau\big) P\big(\overline{X}_\tau\big) \tag{7.12}$$

The repair policy could differentiate damage regions. The first one in the expansion, D_l, contains all damage sizes smaller than those in region 3. If a damage size is in that region it would not be repaired, then the first term in Eq. (7.12) would be zero.

If the damage growth is controlled like that defined in Figure 7.4, then the second term would be zero too. The third term is important, as the specified damage growth definitely makes it the term of focus for the damage resistance design, and the recommended growth rates would make it a threat.

The fourth and fifth terms should be reduced to small values compared to the third through damage resistance design, because they represent the severe to extreme regions.

This discussion makes region 4 the focus of interest and Eq. (7.12) can be reduced to,

$$P(T_\tau) \approx P\big(D_{4\tau}|\overline{X}_\tau\big) \cdot P\big(\overline{X}_\tau\big)$$

The next example, 7.5, will illustrate relative orders of magnitude for the situation described in all examples.

Example 7.5: Eq. (7.9) is the focus of this description. Using the analogous orders of magnitude to what has been used in previous examples, results in the following numbers in the order of the reference equation.

The contribution is:

$$10^{-3} \cdot 10^{-2} \cdot 10^{-2} \cdot 10^{-2} \cdot 10^{-2} \cdot 10^{-1} = 10^{-12}$$

which makes it only a minor influence in this case.

Example 7.5 brings up the need to formulate a repair "policy" that is compatible with the total design criteria set, and points out that a flexible policy can make it possible to embrace an approach that allows a controlled way to delay repairs until safety demands.

It also illustrates the importance of damage resistance and damage growth concerns in the design of safe and efficient composite structures.

Chapter 8

Building Block Approach

The building block approach, "BBA," has traditionally been a way to produce design data in a way that represents a structured process to go from coupons to element to sub-components to panels to PSEs to full-scale components to total airplane testing. What should be accounted for in the allowable values and design data columns and what should be accounted for in the columns of the actual structural response due to variations, flaws, damage, loads and environments? This is an often asked question that is very important in structural design, and a question that requires a very careful answer in composite design.

Figure 8.1 illustrates the starting point, which simply stated, says, "when the applied stress (strain) in the critical location is equal to the allowable, F, then the allowable panel load, $N = N(F)$," has been reached.

This concept has been extended to include stability critical structure. Figure 8.2 describes a case of a skin–stringer panel.

This situation is more complicated due to an intermediate step to element allowables. The element must represent the critical mode of the structure. It could fail in, crippling, buckling displacement induced "pull-through," debonds, locally induced strength failures, post-buckling strength, etc.

The applied stress could be influenced by scatter in tolerances, assembly mismatches and damages (e.g. excessive clamp-up, etc.) flaws, unintended eccentricities (e.g. shims, etc.), secondary effects (e.g. displacement-dependent response, etc.) resulting in random changes to the results. A more detailed knowledge of the failure mechanism clearly is needed to determine the effects of all these "stress-risers." They can possibly make crippling allowable values, reduced stability allowable values, material strength allowable values, adhesive allowable values, fastener allowable values, etc. the critical factors in producing the design data for the process.

So the idea behind "structural allowables" is to contain all these random effects in the "panel allowables" by insisting on production processes for the building of test elements.

In an analogy with Figure 8.1, we can write $N = N(P)$ and if P represents a B-value then, the design of the PSE (or part of one) would be based on B-value allowables. That has been the prevailing practice in the "aluminum world."

Composites add one more dimension to design data acquisition because of the often occurring damage critical structures, and the fact that ultimate allowables will be determined with some damage included. Figure 8.3 illustrates the situation.

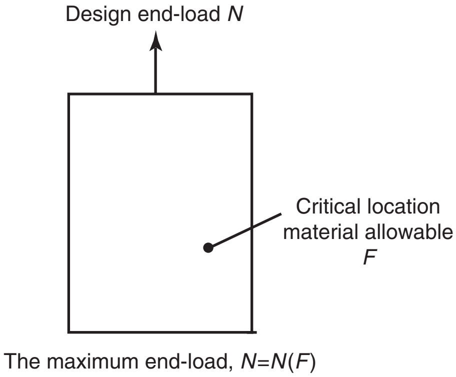


Figure 8.1. Relation between design end-load and material allowables for "material strength critical structure."

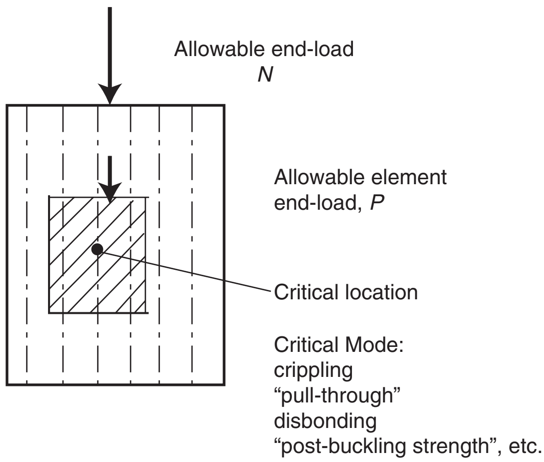


Figure 8.2. Stability critical situation.

The maximum end-load for the panel, $N = N(\mathrm{RS})$ will therefore determine the safety level based on how predictable a "critical damage" is. For situations with structure without service experience, this becomes an uncertainty that has to be reduced by a rational initial position and disciplined "monitoring-processes," and a process for control of safety levels and updating of risk and uncertainty states.

It is also important that the equation

$$N = N(\mathrm{RS})$$

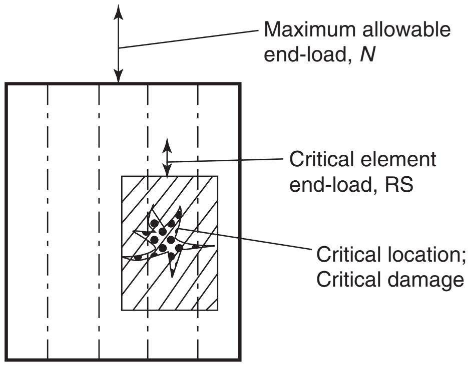


Figure 8.3. Allowable load for a damaged panel.

is based on allowables-quality information for residual strength. Especially for damage critical structure, the probability of failure depends on the definition of the requirement:

$$\Pr(\mathrm{RS} \leq \mathrm{LLR}) \leq p \tag{8.1}$$

where for *B*-value allowables $p = 0.10$, and the maximum allowed end-load would have "*B*-value quality."

8.1. COMPONENTS AND SCALE-UP

Components in the "Composite world" used for allowable determination, design data production, compliance demonstration and "Proof of Structure" can be of many types. The following are potential candidates:

Coupons: Small test specimens for evaluation of basic laminate properties;
Elements: A generic part of a structural member; the "element" includes representations of geometry, scatter in part and sub-assembly geometry, flaw and material properties;
Details: A non-generic segment of a structural member; the "detail" incorporates design specific features;
Sub-components: A significant three-dimensional segment which can provide a complete representation of a section of the full structure; including total loads, scatter in part and sub-assembly geometries and anomalies, and variations in processing;
Component: A major section of the airframe, a complete unit, the test of which can provide "Proof of Structure."

Every entry in the mentioned list represents a different level of size and cost, and traditionally the results from each level have been produced by testing. A natural alternative would aim at having some levels produced by testing and some levels of results be produced by analytical scale-up.

For composites where both ultimate load carrying capability and damage tolerance integrity must be established for a number of different damage types and locations, depending on structural concepts and load levels, this can, and often has resulted in very time consuming and expensive development and design programs. One way to achieve a more efficient and realistic approach to design data, than a "building block approach" is to introduce validated analytical predictions and scale-up from one component level to the next.

An intuitively appealing approach is to conduct allowables testing on the coupon level, produce the "element" design values by testing, use existing technology to combine results from level one and two (coupons and elements) and use structural mechanics to predict structural design data. The next level, sub-components, can also be designed using analytically predicted design values. Some test validation would be in order either for method verification and/or for result confirmation. Finally "Proof of Structure" cannot be achieved without testing, but there are a large number of ways to combine testing and analytical extensions for different damages and damage locations, environmental effects, different load-cases, load levels, different detail solutions and damage growth. A cautious strategy can preserve safety from level to level, if the statistics from the coupon level and element level are preserved in the scale-up.

8.2. ALLOWABLES

Allowables come in two "flavors," material allowables and structural allowables. Material allowables take into account scatter in material properties caused by process variations in "prepreg" and laminate production and physical differences, like fiber waviness, lay-up imperfections, local variations of resin content, etc.

The structural allowables include, in addition to the material scatter, variation in part and assembly geometry, damage caused by handling and transportation, mismatches produced in assembly, eccentricities due to shim, "secondary" stresses due to "clamp-up."

Design requirements that include damages, in the case of composites both ultimate and limit allowables, additional complication arise from damage size, damage type, type of threats and damage severity.

If we ask the question: "How do we scale-up the statistics between coupons and elements?" Then, it seems that there is one direct alternative. It involves designing tests for elements that are crippling critical, strength critical, stability critical and damage critical and interpret the results so that,

$$\Pr(P \leq N_E) \leq p \quad \text{or} \quad \Pr(\mathrm{RS} \leq F_{\mathrm{RS}}) \leq p$$

for the "detail" level, the analysis (local/global) will use prepackaged models of the different elements and when the stress or strain fields reach N_E or F_{RS} the panel end-load, at that time would be the allowable detail (PSE).

At this point, if a proper test coverage had been produced, one would have the design data to design PSEs (sub-components), if the special approach were test validated. As a consequence, sub-component testing may have become superfluous or significantly reduced. The environmental effects could be obtained on the "element level." And a "New Building Block Approach" could have saved both resources and achieved proper safety levels (e.g. by making $p = 0.10$ in the equation above).

"Proof of Structure" for damage tolerance critical components could potentially result in a very risky, expensive and time consuming activity with catastrophic recovery characteristics for premature failure. A demonstration phase proving that the selected approach can use sub-components to reliably predict failures, followed by a period of sub-component-based validation; "Proof of Structure," would save both time and money.

8.3. CRITICALITY

Equal criticality is often defined as the situation when damage tolerance considerations result in the same "thicknesses" as the static strength sizing does. In a deterministic world, it is for linear structure expressed as,

$$1.5 \cdot F_{\mathrm{RS}} = F_{\mathrm{ult}}$$

where the right-hand-side traditionally is represented by *B*-values. In the more complex states of stress and stain, the earlier equation can be expressed, in terms of interaction, as,

$$R_{\mathrm{lim}}\left(\vec{f}_{\mathrm{lim}}, \vec{F}_{\mathrm{lim}}\right) = R_{\mathrm{ult}}\left(\vec{f}_{\mathrm{ult}}, \vec{F}_{\mathrm{ult}}\right)$$

which makes the analogous point.

In a world of "Innovation," where safety of the design approach has not been proven in service, another way to determine criticality is desirable. If one were to consider a given design, and ask: "When is the probability of losing ultimate strength integrity equal to the probability of losing damage tolerance integrity?" Then the resulting answer could be considered a more general definition of equal criticality and it would give detection of damage its proper role in design. Furthermore, in pertinent situations, other kinds of integrity could be included in this type of quest for a "balanced design."

Regulated airplane designs are created with the requirement that, "Limit load is the largest load expected in service," and ultimate loads are derived as 1.5 (safety factor) times limit loads. Consequently, the probability of loss of ultimate strength integrity in zero-margin of safety, for aluminum structures is,

$$P(s \leq F_B)$$

The situation for composite structure is more complicated, because the ultimate requirement includes damage in the size range D_u, and the analogous probability can be written as,

$$P(\overline{B}_u \overline{X} D_u) = P(\overline{B}_u | D_u \overline{X}) \cdot P(D_u | \overline{X}) \cdot P(\overline{X})$$

The probability of loss of damage tolerance integrity is,

$$P(\overline{B}_l \overline{X} D_5) = P(\overline{B}_l | \overline{X} D_5) \cdot P(D_5 | \overline{X}) \cdot P(\overline{X})$$

and again it is clear that both damage tolerance and ultimate strength for composites depend on damage resistance and damage growth characteristics.

Example 8.1: This example illustrates a special case of equal criticality for composites.

The definitions of the pertinent events are:

$$\overline{B}_u\!: \; s \leq \frac{F_B}{1.5}$$
$$\overline{B}_l\!: \; \mathrm{RS} \leq \mathrm{LLR}$$

and ultimate is critical when,

$$P(\overline{B}_u \overline{X} D_u) \geq P(\overline{B}_l \overline{X}\, \overline{D}_6)$$

The guiding equation is,

$$P(\overline{B}_l | \overline{X} D_5) = P(\overline{B}_u | \overline{X} D_u) \cdot P\frac{(D_u | \overline{X})}{P(D_5 | \overline{X})}$$

or the more general case,

$$P\left(\overline{B}_l \overline{X}\,\overline{D}_6\right) = \sum_{i=1}^{5} P\left(\overline{B}_l \overline{X} D_i\right) = \sum_{i=1}^{5} P\left(\overline{B}_l | \overline{X} D_i\right) \cdot P\left(D_i | \overline{X}\right) \cdot P\left(\overline{X}\right)$$

which can be summarized as,

$$P\left(\overline{B}_u | \overline{X} D_u\right) = \frac{\sum_{i=1}^{5} P\left(\overline{B}_l | \overline{X} D_i\right) \cdot P\left(D_i | \overline{X}\right)}{P\left(D_3 \overline{X}\right)} \tag{8.2}$$

The following basic data will be used in Eq. (8.2). The values from Table 8.1 are used in Eq. (8.2) and the last line is the probability of loss of ultimate integrity, if to achieve equal criticality; given the probabilities for residual strength in different regions.

The $P\left(\overline{B}_u | \overline{X} D_u\right)$ is a function of the coefficient of variation, C_v, and is reported in Table 8.2 for a normally distributed allowable.

So for this case, one finds that "ultimate strength criticality" is very unlikely, except for very "bad" data (high coefficient of variation).

Table 8.1. Probability of loss of integrity and damage size

i	RS	Size	Ult.
3	10^{-3}	10^{-1}	10^{-3}
4	10^{-2}	10^{-2}	10^{-3}
5	10^{-1}	10^{-3}	10^{-3}
$\sum$			$3 \cdot 10^{-3}$

Table 8.2. Probabilities of equal criticality for ultimate and limit

C_v	t	$\Phi(t)$	Note
0.05	−7.53	0	
0.10	−4.20	10^{-5}	
0.15	−3.08	$\sim 10^{-3}$	← critical
0.20	−2.53	$5 \cdot 10^{-3}$	$3 \cdot 10^{-3}$ ↓

8.4. CURRENT PRACTICES

Some current practices for assuring static strength integrity focus on "Worst-case" scenarios and situations, e.g. compression critical structure, the following events are considered,

$$\overline{B}_{\mathrm{uc}}\overline{X}D_u\mathrm{FOMT}$$

where

$\overline{B}_{\mathrm{uc}}$: $s \leq$ cut-off value in compression, COC;
F: Fastener is present;
O: "Open hole" behavior prevails;
M: Saturation moisture content reached;
T: Highest temperature reached.

The probability of this "Worst case," W_{c}, is,

$$P\left(W_{\mathrm{c}}|\overline{X}\right) = P\left(\overline{B}_{\mathrm{uc}}|\overline{X}D_u\mathrm{OFMT}\right) \cdot P\left(T|\mathrm{MOF}\overline{X}D_u\right) \cdot P\left(\mathrm{M}|\mathrm{OF}\overline{X}D_u\right) \cdot P\left(\mathrm{O}|\mathrm{F}\overline{X}D_u\right) \cdot P\left(\mathrm{F}|\overline{X}D_u\right) \cdot P\left(D_u|\overline{X}\right) \tag{8.3}$$

for which the "undamaged" equivalent is,

$$P(W_{\mathrm{c}}|X) = P\left(\overline{B}_{\mathrm{uc}}|X\mathrm{OFMT}\right) \cdot P(T|X\,\mathrm{FOM}) \cdot P(\mathrm{M}|\mathrm{OFX}) \cdot P(\mathrm{O}|\mathrm{F}X) \cdot P(\mathrm{F}|X) \tag{8.4}$$

Both Eqs. (8.3) and (8.4) could be interpreted as the probability of loss of structural integrity (in this case static strength integrity) and Example 8.2 illustrates orders of magnitude.

Example 8.2: Starting with Eq. (8.3), the following assumptions for the factors on the right-hand side are made,

The second factor: 10^{-2};
The third factor; assuming toward the end of "life": 1;
The fourth factor; assuming consideration for "bolted repair": 0.10^{-1};
The fifth factor: 10^{-1};
The sixth factor: 1.

The estimates presume a situation late in the operational life of a compression critical PSE. The probability used in the previous examples for loss of integrity would then yield,

$$P(\overline{B}_{\mathrm{uc}}|\overline{X}D_u\mathrm{OFMT}) \leq 0.10$$

This indicates that even damaged "open-hole" panel assumptions must be considered for use of *B*-value allowables for local designs, and *B*-values for the maximum end-loads.

The analogous arguments applied to Eq. (8.4) would result in,

$$P(\overline{B}_{\mathrm{uc}}|X\mathrm{OFMT}) \leq 0.01$$

Using worst-case scenarios in the design process must be done with caution. Intuition is often less than perfect in these complex design contexts. It also seems very severe to combine open-hole and damage as in Eq. (8.3).

Current practices for the design of tension critical structures often deals with the following two "worst-case" scenarios,

$$\overline{B}_{\mathrm{ut}}\overline{X}D_u|\mathrm{F} \quad \text{or} \quad \overline{B}_{\mathrm{ut}}\overline{X}D_u|\overline{\mathrm{F}}$$

where

$\overline{B}_{\mathrm{ut}}$: $s \leq$ cut-off value for ultimate tension, COT;
F: Fasteners are involved at the location.

The situation is based on the often present characteristic that tension is insensitive to moisture and high temperature, which would mean that room temperature properties could be "driving" the design.

The expansion in probabilities is,

$$P(\overline{B}_{\mathrm{ut}}\overline{X}D_u|\mathrm{F}) = P(\overline{B}_{\mathrm{ut}}|\overline{X}D_u\mathrm{F}) \cdot P(D_u|\overline{X}\mathrm{F}) \cdot P(\overline{X}|\mathrm{F}) \tag{8.5}$$

and analogously for the "no-fastener-case."

Example 8.3: If we concentrate on a tension panel with fasteners in the critical location, the following values could be applicable,

The second factor: 10^{-1};
The third factor: 10^{-3}.

Resulting in,

$$P(\overline{B}_{\mathrm{ut}}|\overline{X}D_u\mathrm{F}) \leq 0.10$$

Where it has been assumed that an accidental damage at the critical location of a fastener line is a fairly unusual event, which depends on location. This has been used in some applications. It clearly is a criterion that has to be justified from case-to-case and requires insight in $P(\overline{X}|\mathrm{F})$ (probability of damage at fastener locations), and certainly would involve considerable uncertainty. So any attempt to generalize the advantage in this approach must be justified in detail.

8.5. FACTORS OF SAFETY

Limit load is:

"The largest load expected in service."

Consequently, the probability,

$$\Pr\left(s \leq \frac{F_B}{n}\right) \tag{8.6}$$

where n is the ultimate factor of safety, is an expression for the probability of loss of limit integrity and equivalent to the:

"probability of failure during a lifetime."

The following example demonstrates orders of magnitude for a normally distributed strength, s.

Example 8.4: The objective of this example is to display probabilities associated with "limit strength" and B-value basis.

B-value ultimate allowables are the baseline, and the assumption of "Normal distribution," supports the following equation,

$$\Pr(s \leq F_B) = \Phi\left(\frac{F_B - \mu}{\sigma}\right) = \Phi(t) \tag{8.7}$$

where $\Phi(t)$ is the distribution function for the standardized normal distribution. The B-value assumption yields,

$$\Phi(t) = 0.10 \Rightarrow t = -1.30 \Rightarrow \frac{F_B}{\mu} = 1 - 1.30 \cdot C_{\mathrm{v}}$$

where C_v is the coefficient of variation. We now substitute the results into Eq. (8.6),

$$\Phi\,\frac{(F_B/n\mu)-1}{C_v}=\Phi\,\frac{(1-n/C_v)-1.30}{n}$$

Table 8.3 shows some parameter variation of n, which would apply to the factors of safety.

The effects of margin of safety (classical definition) on the probabilities is described in Table 8.4. The following nomenclature used,

Margin of Safety: MS;
Safety Factor: SF;
Equivalent Safety Factor: $SF_e=(1+MS)\cdot SF$.

This special case applies to $SF=1.5$. An extended discussion can be found in Appendix A.

There are two trends in situations like this, which are worth mentioning,

1. Reduction of the safety factor very quickly results in much increased probability of failure;
2. Relatively minor increases in margin of safety (e.g. 0.10) reduce probability of failure significantly.

Table 8.3. Factors of safety and probability of failure in one life

	$C_v=0.10$		$C_v=0.15$		$C_v=0.20$	
n	t	Φ	t	Φ	t	Φ
1.5	−4.20	10^{-5}◀	−3.08	0.001	−2.53	0.006
1.4	−3.79	$7\cdot 10^{-5}$	−2.83	0.002	−2.35	0.009
1.3	−3.31	$5\cdot 10^{-4}$	−2.57	0.005	2.15	0.016

Table 8.4. Margin of safety vs probability of failure

		$C_v=0.10$		$C_v=0.15$	
MS	SF_e	t	Φ	t	Φ
0.10	1.65	−4.73	10^{-6}◀	−3.41	0.0003
0.20	1.80	−5.17	0	−3.68	0.0001
0.30	1.95	–	0	–	–

A successful scale-up in a ''New Building Block Approach'' requires a thorough understanding of modes of failure and failure mechanisms. A successful design identifies the critical failure mechanisms, controls the local design through allowables or critical stress intensity factors and relates maximum end-loads to the load level that causes the local stresses (strains) to reach the critical level.

Chapter 9
Design Scenarios

The design of damage tolerance critical composite structures is inherently more complicated than the practices developed for metal structure. Two different routes are taken. In the commercial world, static strength is dominating the acreage of structure and fatigue rules for the detail designs. Residual strength and crack growth (damage tolerance) mostly play a role in the selection of inspection methods and intervals. However, mostly in the military, damage tolerance plays a significant role in the design and then often introduced as a modification of the ultimate structural allowables (undamaged allowables corrected for the effects of damage and damage growth).

The typical damages introduced and used for the design of metal structure are based on cracks. Figure 9.1 describes the damages that are considered for riveted aluminum skin–stringer tension critical structure.

A PSE consisting of a part of, e.g. a wing lower surface of a commercial airliner would be designed for these types of damage assuming limit load for the overall damage tolerance requirements. This is typically the case for so called ‘‘Fail-Safe structure’’ where in the presence of one of these three damages (among other requirements) the remaining structure can redistribute and carry all the loads.

All these damages consist of a ‘‘failed element’’ and a partial failure of the skin. This type of damage tolerance justifies the use of *B*-value allowables in achieving ultimate strength. The design world of composite structure presents a considerably more complicated picture. Figure 9.2 describes types of damage that has been occurring and can be expected to occur in a composite structure.

While the metal example is dominated by tension cracks (fatigue is perceived as the main threat; it could be debated that corrosion deserves the same attention), composite skin–stringers are sensitive to damage in compression structure and in tension and mainly caused by accidental damage.

Figure 9.2 shows five types of damage for skin–stringers, and honeycomb, e.g. would add other types. Part of good damage tolerance designs, no matter what concept is involved, should be based on a thorough knowledge of threats (environments) and damage types.

The five types of damage are:

1. Debonds;
2. Cracks;
3. Broken fibers and matrix cracks;

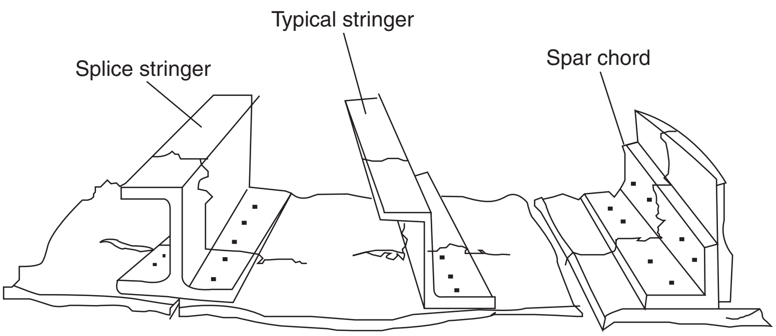


Figure 9.1. Typical damages in an aluminum skin–stringer panel.

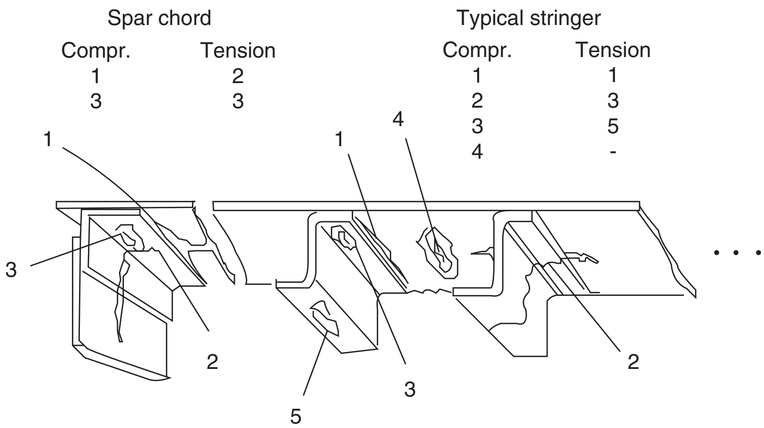


Figure 9.2. Damage types for composite skin–stringer construction.

4. Delaminations;
5. Flange impact damage.

Several damage types are applicable to both compression and tension situations, which is important for many PSEs, as load reversal is more often the rule than the exception.

9.1. DAMAGED METAL STRUCTURE

This example focuses on a PSE that represents a significant part of an aluminum lower surface of a wing. Figure 9.1 addresses three typical types of damage locations, typical stringers, splice stringers and spar chords.

This case involves m typical stringers, i, two splice stringers, ss, and two spar chords, sc, and deals with a structure that is equally critical in ultimate static strength and in damage tolerance. The loss of damage tolerance integrity can be expressed as:

$$P(\overline{U}) = \sum_{i=1}^{m} P(\overline{B}_i \overline{X}_i D_{5i}) + 2 \cdot P(\overline{B}_{ss} \overline{X}_{ss} D_{5ss}) + 2 \cdot P(\overline{B}_{sc} \overline{X}_{sc} D_{5sc}) \tag{9.1}$$

Example 9.1 addresses the orders of magnitude of probabilities in the metal world.

Example 9.1: Today's practice in the "metal world" includes using the mean value of tests of residual strength as the allowable for damaged structure. If we focus on a typical term of the sum on the right-hand side of Eq. (9.1), the following values are reasonable (mainly results of definitions),

$$P(\overline{B}_i \overline{X}_i D_{5i}) = P(\overline{B}_i | \overline{X}_i D_{5i}) \cdot P(D_{5i} | \overline{X}_i) \cdot P(\overline{X}_i) = 0.50 \cdot 10^{-3} \cdot 10^{-1} = 0.5 \cdot 10^{-4}$$

Static strength integrity can be assumed to be lost, e.g. when the strength is less than $F_B/1.5$, which for normally distributed variables it approximately happens with a probability,

$$\Pr\left(s \leq \frac{F_B}{1.5}\right) \approx 10^{-4}$$

So, the probabilities of "the loss of static strength integrity" and "the loss of damage tolerance integrity" are of the same orders of magnitude, for this kind of typical aluminum skin–stringer. So this example illustrates some aspects of the practices in the metal world.

These practices have evolved hand-in-hand with aluminum materials and processes developments that have produced an empirically mature and balanced design approach that only can be replaced successfully by a composites design process, if the objective continues to be to match or exceed the level of safety of metal structure it replaces.

Figure 9.2 illustrates a typical set of composite damage types. The variety of types certainly makes damage tolerance for composites much more complicated than what current metal practices have resulted in for the "metal world."

9.2. DAMAGED COMPOSITE STRUCTURE

The many types of damage, are important for loads other than tension, and the susceptibility to accidental damage are concerns in the design of damage tolerance critical composite structure. The typical stringer in Figure 9.2 is potentially exposed to four types of compression critical damage and three types of tension critical damage.

Any kind of respectable design process must include a phase of criteria development that establishes the "likely" accidental damage threats and the requirements on probabilities of damage sizes and probabilities of damage growth rates. Accidental damage is a very important consideration in design (even demonstrated in the latest space shuttle disaster), and a thorough evaluation is a must.

Example 9.2: This example focuses on a composite compression skin–stringer panel, e.g. a wing upper surface. It is assumed to contain two types of elements; typical stringers and spar chords (alternative design solutions are assumed for vent and splice stringers).

The probability of loss of damage tolerance integrity can be written as,

$$P(\overline{U}) = \sum_{j=1}^{n} \sum_{i=1}^{4} P_j(\overline{B}_j|\overline{X}_i D_{5i}) \cdot P_j(D_{5i}|\overline{X}_i) \cdot P_j(\overline{X}_i) + 2 \cdot \sum_{i=2}^{3} P(\overline{B}_{sc}|\overline{X}_i D_{5i}) \cdot P(D_{5i}|\overline{X}_i) \cdot P(\overline{X}_i)$$

where the index j is a typical stringer and sc is the spar chord.

If we assume equal contribution from stringers and spar chords and from compression and tension (e.g. from 1 g loads), we have:

$$P(\overline{U}) \approx 100 \cdot P(\overline{B}|\overline{X}D_5) \cdot P(D_5|\overline{X}) \cdot P(\overline{X})$$

If we now return to the orders of magnitude used as baseline for the previous examples we have,

$$P(\overline{S}) = 10^{-9} \geq 10^{-5} \cdot P(\overline{U}) \Rightarrow P(\overline{U}) \leq 10^{-4}$$

We now postulate a damage resistant design with modest damage growth rates, and design data that supports,

$$P(D_5|\overline{X}) = 10^{-3}$$

If we in addition notice that the situation in hand applies to the situation just after a major inspection: "damage is present at T, T_1, T_2," etc. Then we can estimate,

$$P(\overline{X}) = 10^{-2}$$

The resulting requirement for residual strength is,

$$10^{-4} = 100 \cdot P\left(\overline{B}|D_5\overline{X}\right) \cdot 10^{-3} \cdot 10^{-2} \Rightarrow P\left(\overline{B}|D_5\overline{X}\right) = 10^{-1}$$

which yields a residual strength requirement for damage region 5; EDD $< D_s <$ MAD, that is a *B*-value.

This is an example, but it shows that the design criteria development is a very important part of the design of any PSE, and that requirement definition has to be based on the required level of safety, because it is "in no way" given.

9.3. DAMAGE CRITERIA

The selection of damage sizes for "Ultimate design requirements," "Limit (damage tolerance) design requirements and realistic threats must be based on an intricate balance between 'damage resistance,' damage growth rates" (in the critical environment), selected inspection methods, life cycle cost, weight and required safety levels. The basic controlling variable is residual strength, and Figure 9.3 presents a view of the "allowable requirement" over the design space.

There are two regions of design critically; where the "Ultimate static strength" requirements prevail (for a PSE) and where the "Damage tolerance" requirements are dominating. Then there is the question: "Why does not the designer choose an approach that makes them both equally critical?"

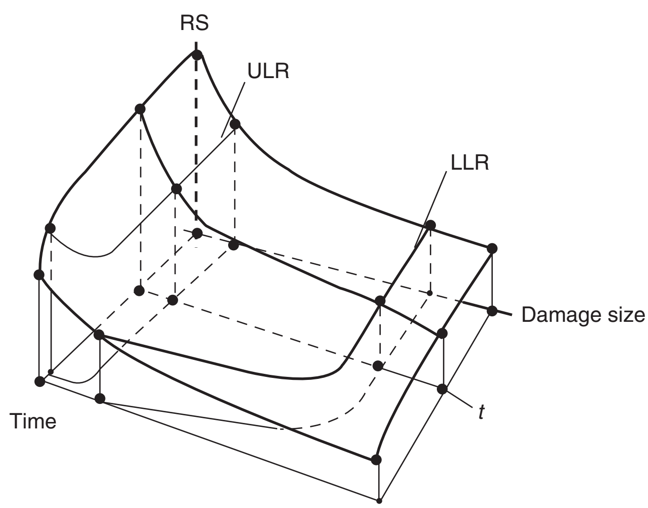


Figure 9.3. Residual strength allowables space.

Figure 9.4 describes a "Damage tolerant" critical situation at the time *t* indicated in Figure 9.3.

The design variables (e.g. thicknesses) are determined by limit loads considerations (the lowest allowables comes from damage tolerance requirements). Obviously, the ultimate design could be based on a larger damage size, so that maximum ultimate damage, "MUD" is made equal to the "equally critical damage" size, "ECD," provided that the "non-detectable damage," "NDD" is smaller. In which case the design could be performed using "Static Ultimate Requirements." Alternatively, the static ultimate strength sizing could be done to a margin of safety that compensates for the difference.

Figure 9.5 describes an ultimate strength critical situation. The alternative illustrated in the figure is tempting to change by: "Reducing MUD so it becomes equal to ECD." A more prudent alternative might be to increase MUD until ECD becomes equal to MUD. Figure 9.6 illustrates how inspection characteristics "enter into the picture."

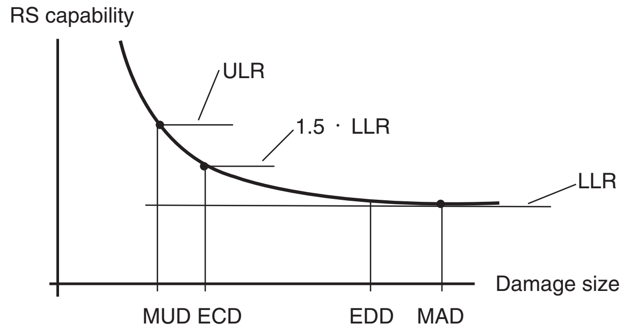


Figure 9.4. Damage tolerance critical design.

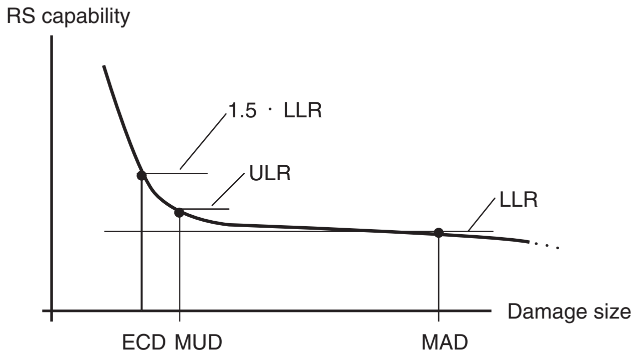


Figure 9.5. Ultimate static strength critical situation.

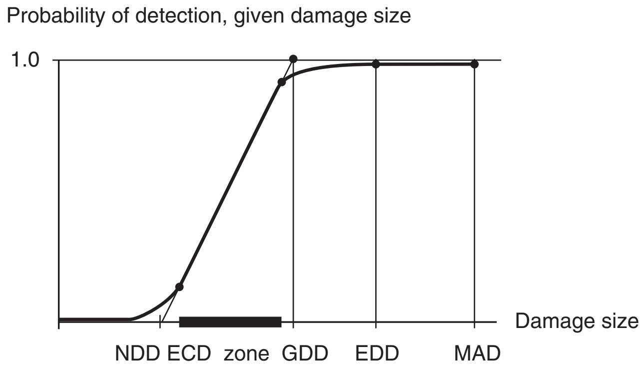


Figure 9.6. Inspection quality in terms of damage sizes.

The probability of detection in region 5 (EDD, MAD) is a very important aspect of the probability of "Safe Flight." Demonstrated orders of magnitude in the previous examples have indicated that a realistic target for the Probabilities of Non-Detection of EDD and MAD have practically "come out" as,

$$P(\overline{H}|\mathrm{EDD}) \ll 10^{-2} \quad \text{and} \quad P(\overline{H}|\mathrm{MAD}) \equiv 10^{-3}$$

and illustrates the potential constraints that inspection methods puts on the prudent selection of damage tolerance maximum damage sizes.

For situations when the slope of the curve for residual strength in region 5 still is significant, a set of sub-regions must be defined so the precision of predicted residual strength and growth rates can be held within requirements.

Figure 9.6 shows the ECD zone where one can choose to make damage tolerance and ultimate static strength equally critical, achieving the advantage using "true and tried" methods for design. It also avoids a damage tolerance allowables-program. The approach for potentially damage tolerance critical structure could be focused on a demonstration of equal criticality, by showing that,

$$\text{Ultimate Strength} = 1.5 \cdot \text{LLR}$$

provided both satisfy some strict allowables-requirement and is part of the compliance demonstration.

So with this approach to design, one could pursue the same methods for the two possible situations illustrated in Figures 9.4 and 9.5.

One way to define criticality in allowable-based (safety-based) designs is to compare probabilities of violating different kinds of integrity.

Recognizing that the largest expected load under safe operating conditions is limit load, we will use the limit case as a baseline.

The probability of violating ultimate integrity, U_{u}, is:

$$P(\overline{U}_u) = P(\overline{B}_{\mathrm{u}}|D_2\overline{X}) \cdot P(D_2|\overline{X}) \cdot P(\overline{X}) \tag{9.2}$$

The following definitions apply,

F_B: B-value allowable;
$\overline{B}_{\mathrm{u}}$: $s \leq (F_B/1.5)$;
s: Strength;
D_2: NDD $< D_{\mathrm{s}} <$ MUD.

The probability of violating damage tolerance integrity, U_l, is,

$$P(\overline{U}_l) = P(\overline{B}_l|D_5\overline{X}) \cdot P(D_5|\overline{X}) \cdot P(\overline{X}) \tag{9.3}$$

The participating events are:

$\overline{B}_l$: $s \leq F_{\mathrm{RS}}$;
D_5: EDD $< D_{\mathrm{s}} <$ MAD

The following inequality, if satisfied, assures ultimate criticality:

$$P(\overline{U}_{\mathrm{u}}) \geq P(\overline{U}_l) \tag{9.4}$$

Example 9.3 illustrates the numerical consequences for orders of magnitudes in the "practical" range, and deals with the requirements to satisfy to assure ultimate criticality, or if preferred, the basis for margin of safety of damage tolerance.

These considerations could be an important part of establishing the damage regions. Figure 9.7 shows an example of the definitions of damage regions that could be used in the design, the design criteria development and the selection of inspection procedures.

The graphs for detection and residual strength are typical. For the cases, where deviations from the "typical" situation are present, additional considerations for the rational choice of regions may be needed. Example 9.3 includes some values that indicate potential requirements.

Example 9.3: This example is an illustration of criticality and the role of structural integrity in the design process. But we start with a few thoughts about Figure 9.7.

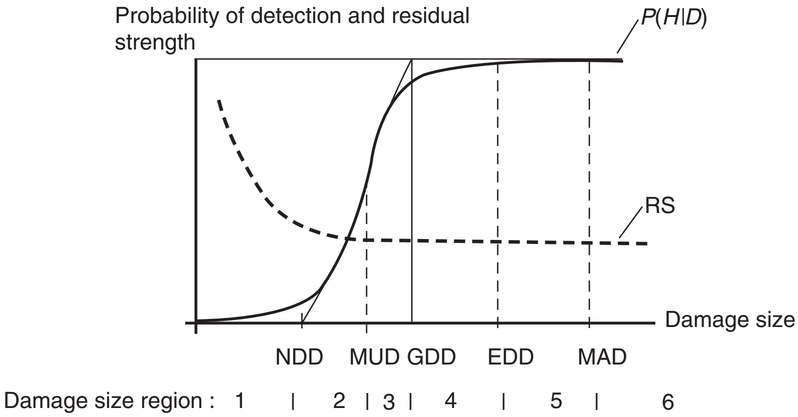


Figure 9.7. Typical damage size regions.

One way to approach the selection process is to establish a few trial fixed points, which, as an example, could be subject to the following definitions of probability of detection:

$P(H|\mathrm{NDD}) < 0.05$;
$P(H|\mathrm{GDD}) > 0.95$;
$0.999 > P(H|\mathrm{EDD}) \gg 0.99$;
$P(H|\mathrm{MAD}) > 0.999$.

The next step could involve inequality (Eq. (9.4)), which could be rewritten as,

$$P\left(\overline{B}_u|D_2\overline{X}\right) \cdot P\left(|D_2|\overline{X}\right) \geq P\left(\overline{B}_l|D_5\overline{X}\right) \cdot P\left(D_5|\overline{X}\right)$$

And returning to orders of magnitude used in the previous examples, we would find,

$$P\left(\overline{B}_u|D_2\overline{X}\right) \cdot 10^{-1} \geq P\left(\overline{B}_l|D_5\overline{X}\right) \cdot 10^{-3}$$

The expression can lead to the following conclusion, if we introduce the demonstrated behavior of normally distributed variables and a *B*-value based ultimate allowable,

$$P\left(\overline{B}_l|D_5\overline{X}\right) \leq 10^2 \cdot P\left(\overline{B}_u|D_2\overline{X}\right) = 10^{-1} \tag{9.5}$$

which represents a *B*-value requirement for residual strength in the "large" damage region.

It is hard to imaging relieving the ultimate *B*-value requirement, so the "trading" room is centered around the flexibility of EDD and to some degree around MAD. An involvement of alternative inspection approaches in the process would then make it possible to trade both EDD and MAD versus design data quality and damage resistance criteria.

Suppose that we now ask what is critical for a given thickness and for specific values of the coefficient of variation for the residual strength. We now choose a different version of inequality (Eq. (9.4)),

$$\Pr\left(s \leq \frac{F_\mathrm{u}}{1.5}\right) \geq 10^{-2} \cdot \Pr(s \leq F_\mathrm{RS})$$

where

F_u: Ultimate allowable;
F_RS: Residual strength allowable.

This inequality represents the condition that makes ultimate strength critical.

Choosing standard practice in the metal design world yields the following condition,

$$\Pr\left(s \leq \frac{F_\mathrm{u}}{1.5}\right) \geq 0.005$$

and the investigation, assuming a normally distributed variable, shows that a sufficient condition for satisfying the inequality is that,

$$C_\mathrm{v} < 0.15$$

The same result applies to be true for *B*-value residual strength allowables, for which the inequality is

$$\Pr\left(s \leq \frac{F_\mathrm{u}}{1.5}\right) \geq 0.001$$

The condition also requires that "larger" allowables than what the *B*-value definition requires, so the prudent design approach would satisfy the inequality (Eq. (9.5)) for the safety target (one unsafe flight in 100 000), being used in this series of examples.

And damage tolerance critical structure needs a very detailed evaluation of what type of allowables should be used for residual strength.

9.4. STRUCTURAL ALLOWABLES

Development of design data for safe composite structural design raises the question of sources of variability, especially of strength and stiffness data. Conventional material allowable values account for scatter in "basic" material properties, but does not capture more than a limited number of random behaviors. The scatter and uncertainties involving laminate properties starts with the variations in "prepreg," tow, etc., originating in sizing of fibers, resin content, fiber types, fiber waviness, processing, postcure, just to mention a few. It is the objective of most material allowable-programs to capture these effects. However, the path from materials to design details and structural concepts (like skin–stringers, honeycomb, etc.) is often lined with many additional sources of randomness and uncertainty.

While detail designs mostly have to be dealt with on a case-by-case basis, structural concepts lend themselves well to a structural allowable process. Additional sources of random variations can then be accounted for in element and "panel" testing. Statistical, structural allowable values can incorporate the random effects. The uncertainty often must be left to the monitoring of service data to be resolved. The worst-case scenarios that have been considered in the metal design world often are impractical for composites.

For example, environmental effects (temperature, moisture, corrosive interaction between dissimilar materials, etc.) can have large influence on the behavior, but how they should be combined with other uncertain phenomena like creep and relaxation and their relief of "built-in" responses, not to speak of the interaction with damage effects, damage growth rates and damage resistance.

There is a number of quantifiable effects emerging in the part and sub-assembly processing, like co-curing of skins and stringers, fastener installations, secondary bonding, thickness variations in part production, mismatches in fit-up, geometry variations, tolerance build up, shimming, flaws in part and assembly, among others. Many of theses effects are process dependent and need case-to-case assessments.

The composite design requirements include, even for ultimate integrity, considerations of the effects of hard-to-detect damage sizes. In principle, there are two types of damage, manufacturing flaws and accidental damage inflicted after start of service. The examples shown in Figure 9.2 could be of either type.

One way to define ultimate allowable requirements would be to relate them to "ultimate structural integrity" by asking the question: "What is the probability of loss of ultimate integrity of a PSE?" The following equation could be a description of the state in Figure 9.2,

$$\Pr(s \leq \mathrm{ULR}) = \sum_{i=1}^{n}\sum_{j=1}^{m} P\left(\overline{B}_{ij} D_{2ij} \overline{X}_{ij}\right), \text{ where} \tag{9.6}$$

$\overline{B}_{ij} = s_{ij} \leq \mathrm{ULR}_{ij}$, the load carrying capability at stringer i with damage at location j is inadequate

A typical term for stringer i is,

$$P\left(\overline{B}_{ij} D_{2ij} \overline{X}_{ij}\right) = P\left(\overline{B}_{ij} | D_{2ij} \overline{X}_{ij}\right) \cdot P\left(D_{2ij} | \overline{X}_{ij}\right) \cdot P\left(\overline{X}_{ij}\right) \tag{9.7}$$

If the question: "Are all the stringer critical for some ultimate load case?" is answered "yes," then one can rewrite Eq. (9.6) as,

$$\Pr(s \leq \mathrm{ULR}) = n \cdot \sum_{j=1}^{m} P\left(\overline{B}_{j} D_{2j} \overline{X}_{j}\right) \tag{9.8}$$

The typical term is,

$$P\left(\overline{B}_{j} | D_{2j} \overline{X}_{j}\right) \cdot P\left(D_{2j} \overline{X}_{j}\right) \tag{9.9}$$

Example 9.4 shows a few situations based on a normally distributed residual strength and the requirement that,

$$\Pr\left(s \leq \frac{F_{\mathrm{u}}}{1.5}\right) \leq \Pr(s \leq F_{\mathrm{RS}})$$

where F_{u} is the ultimate allowable and F_{RS} is the damage tolerance allowable.

Example 9.4: We return to the orders of magnitudes used in the previous examples, which will set the following bound,

$$P\left(\overline{U}\right) \leq 10^{-5}$$

We now assume that there is one dominating damage type (in 15 stringers)

$$10^{-5} \geq n \cdot P\left(\overline{B}_{j} | D_{2j} \overline{X}_{j}\right) \cdot P\left(D_{2j} \overline{X}_{j}\right) = 15 \cdot P\left(\overline{B}_{j} | D_{2j} X\right) \cdot 10^{-2}$$
$$\Rightarrow P\left(\overline{B}_{j} | D_{2j} \overline{X}_{j}\right) \leq 0.67 \cdot 10^{-4}$$

and the resulting value of t in the normal distribution, $\Phi(t) = 0.07 \cdot 10^{-4}$

$$\frac{F_{\mathrm{u}}}{\mu} = 1.5 \cdot (1 - C_{\mathrm{v}} \cdot 3.81) = 0.93 \quad \text{for } C_{\mathrm{v}} = 0.10,$$

$$\text{which gives } t = \frac{(F_{\mathrm{u}}/\mu) - 1}{C_{\mathrm{v}}} = -0.7$$

$$\text{if } n = 10$$

$$\text{then } t = -1.4 \Rightarrow \Phi(-1.4) = 0.08$$

It is striking how the number of types of damage, the number of stringers and the coefficient of variation establish a very sensitive situation in the search for a practical requirement for the ultimate allowable value.

However an attempt to satisfy the inequality,

$$\Pr\left(s \le \frac{F_{\mathrm{u}}}{1.5}\right) \le \Pr(s \le F_{\mathrm{RS}})$$

would reduce the allowables program to damage tolerance allowables, which at least is both relief and support of safety at the same time.

If on the other hand, one wants to establish an equivalent B-value ultimate design value, then Example 9.5 illustrates how a damage tolerance critical structure can be sized for allowable ultimate data based on "safety factors" greater than 1.5.

Example 9.5: We assume, in agreement with orders of magnitude in the previous examples, that:

$$\Pr(s \le F_{\mathrm{RS}}) = 10^{-4} \Rightarrow \frac{F_{\mathrm{RS}}}{\mu} = 1 - 4.25 \cdot C_{\mathrm{v}}, \text{ which for } C_{\mathrm{v}} = 0.10$$

and a safety factor 1.5 yields

$$\Pr(s \le F_{\mathrm{u}}^{\mathrm{equ}}) = 0.001 \Rightarrow F_{\mathrm{u}}^{\mathrm{equ}} = 0.69\mu$$

and the equivalent B-value is, $F_B^{\mathrm{equ}} = 0.87\mu$ yielding a "safety factor" of 1.89, and an equivalent B-value for $C_{\mathrm{v}} = 0.10$ of,

$$F_B^{\mathrm{equ}} = 1.89 \cdot F_{\mathrm{RS}}$$

and the traditional ultimate load design could be executed, if the safety factor equivalent can be determined provided that the coefficient of variation is known.

The example shows that B-value quality is totally driven by the safety requirements, if the structure (PSE) is damage tolerance critical, and there is considerably more to criticality than a comparison between allowable values.

9.5. LIMIT LOADS REQUIREMENTS

Design loads are in detail spelled out in the international regulations (FAR and JAR). The regulations also define what a limit load is and it says:

"A limit load is the largest load expected in service."

Figure 9.8 shows a typical internal flight loads picture for a balanced design of a PSE. Applying the regulation to a distinct definition has many versions, but design requires that it extends to internal loads which may include responses that are not included in the definition of flight load cases, e.g. "built-in" loads caused by assembly mismatches. If we interpret Figure 9.8 as a depiction of *total* internal loads, one interpretation for a service objective of n flights could be for the maximum, given "safe operation,"

$$\Pr[\mathrm{Max}(N_i^{\max}) \leq \mathrm{LL}] = 1$$

The probability that the largest maximum value does not exceed limit load, given "safe operation" is equal to one, and analogously for the absolute values of the minima.

Example 9.6 illustrates potential ways to interpret probabilities involved when "safe operation" is given (see Chapter 1).

Example 9.6: We assume an airplane with 30 000 flight service objective. If we assume that the probability of reaching limit load during an arbitrary flight,

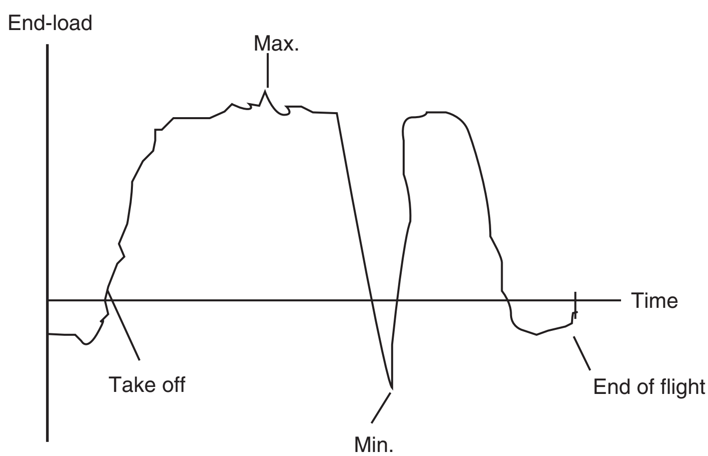


Figure 9.8. End-loads during one flight.

$p_L = 0.33 \cdot 10^{-5}$, then the probability of reaching limit loads k times during 30 000 flights is:

$$\binom{n}{k} \cdot p_L^k = p_k$$

which is,

k	p_k
1	1
2	1/2
3	1/6
4	1/24
.	
.	
.	

and could be used as a priori probability description. The issue of a priori probability density, $p(\mathrm{l}|\mathrm{O})$ functions for internal loads would then be based on a consistent assumption. Figure 9.9 shows a set of contenders.

Where the critical end-load, $N = k \cdot \mathrm{LL}$, occurs when $k = 1$. There is a linear candidate, an exponential and an extreme value distribution. The figure illustrates

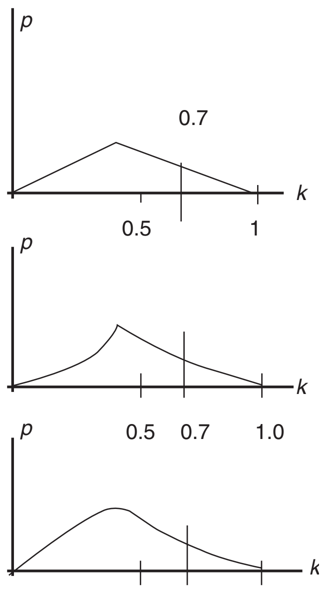


Figure 9.9. Distributions of end-loads.

the point that the "tail" between 0.7 and 1 is an important characteristic in assessing survivability with lost damage tolerance integrity, but maybe with preserved "get-home" integrity. These distributions could be a set of useful tools in assessing the damage tolerance requirements in parametric form, in combination with inspection definitions.

9.6. "NEW" STRUCTURAL CONCEPTS

Skin–stringer constructions have dominated the aluminum world. Composites have not only opened the "structural concepts market" (honeycomb sandwich, stitched RFIs, etc.) but have added variations to the skin–stringer idea with options like co-cured stiffeners, secondary bonded stiffeners, mechanically fastened stiffeners, stitched stiffeners, etc. all of which have different characteristics, with regard to damage resistance, damage growth and damage tolerance, and in addition displaying different modes of failure and failure mechanisms.

Consequently, there is a steady stream of uncertainties, even when materials and processes have stabilized and produced service experience. Uncertainties must be reduced in the design phase and then later monitored in service to guard against surprises. "History" has also given us a gallery of "new" damage threats and maybe a new view of requirements for accounting for accidental damage in designing safe structures.

We have seen an increase in incidents that involves construction debris. Hail impact by large hailstones during flight has been repeatedly reported. Tire fragments from landing gear, tires bursting in flight have caused considerable damage. Unreported collisions with ground vehicles continue to cause destruction. Undetected damage from turbine disc fragments from disintegrating engines has been discovered long after the event took place.

It seems that accidental damage threats have become a very important part of structural design, and that design criteria (and regulations) should spell out the threats in terms of type, size and load requirements. Especially designed tests to determine what damage types and sizes "new" structural concepts exhibit when exposed to realistic threats. If nothing else, it would be a way to determine maximum damage sizes for different levels of damage resistance. It would also be the baseline for realistic damage tolerance criteria. All these measures would help the design community to deal with safe innovation.

It also seems that the variety of damage types begs an approach to the definition of damage that can be of criteria type and can serve as an envelope for damage tolerance requirements. It would serve as the standard for the design and it would assure that damage types and sizes less critical would not violate integrity. It would be demonstrated that the maximum "real world" damage for each type, would not be more critical than criteria damage. The resulting "limit load requirement," LLR $= F_{RS} \cdot \bar{t}$, can be used as a match with either N_{max} or N_{min}. Example 9.7 shows numerical orders of magnitude.

Example 9.7: Figure 9.10 shows the typical situation in the use of criteria damage, where a basic damage tolerance allowable is statistically based. We assume two normal distributions, $n(\mu_i, \sigma)$ and $n(\mu_c, \sigma)$,
The example deals with the following joint event,

$$P(BF) = P(B|F) \cdot P(F)$$

where the participating events are:

$B \leftrightarrow \mathrm{RS} > F_{RS}$;
$F \leftrightarrow F_{RS} > F_a$.

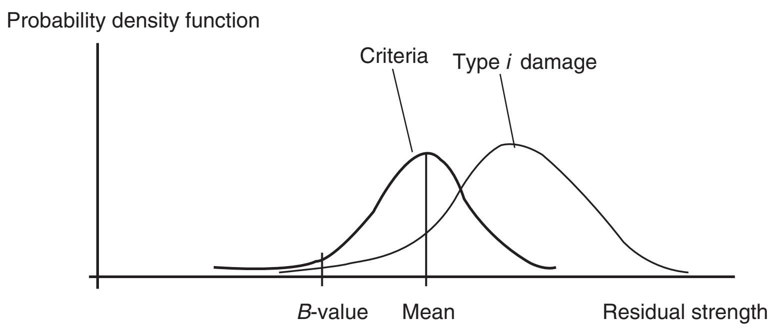


Figure 9.10. Comparison; criteria to realistic damage.

Table 9.1. Probability of being below criteria damage value

k	t	$\Phi(t)$
	B-value	
1	−1.3	0.10 ←
1.1	−2.3	0.01
1.2	−3.3	0.004
1.3	−4.3	0.000008
	Mean	
1.1	−1	0.16
1.13	−1.30	0.10 ←
1.2	−2	0.02

If we consider BF an acceptable design criterion, then we have for the two cases $F_a = F_B$ and $F_a = \mu$ with $C_v = 0.10$. We start the investigation by asking: "What is the probability that the residual strength, RS, for the ith damage type is less than the criteria value?" The ratio between the mean of the criteria case and the ith case is k. The answer is: If the mean is used for the criteria damage and a probability of 0.10 (B-value) is wanted for the ith damage type its mean must be 13 per cent higher than that of the criteria damage, if the coefficient of variation is 0.10. Table 9.1 shows a set of probabilities that could be appropriate in different safety situations.

A thorough criticality evaluation of damage threats and the definition and demonstration of a most critical, realistic "criteria damage" could be an efficient approach to composite, damage tolerance design. The concept of "criteria damage" should be evaluated and embraced wherever practical.

Chapter 10
The Design Process

The design of composite structures, needs a different approach than what is used for metal structure. It needs a process that is homed in on innovation and safety as explicit requirements, and which is adaptable to a "never-ending-stream" of new materials, processes and structural concepts. It must promote explicit safety requirements, compensate for the frequent lack of service experience, and center on "Safe Flights."

The probability of a safe flight involves the probability of preserved structural integrity, which is a very important aspect of structural design. The probability can be written for damage tolerance critical structure as,

$$P(U) = P(B_l X) + P(B_l \overline{X}) \tag{10.1}$$

where the first term on the right-hand side represents "aging." For cases without degradation the probability of lost integrity can be written as,

$$P(\overline{U}) = P(\overline{X}) \cdot \left[P\left(\overline{B}_l | \overline{D}_6 \overline{X}\right) + P\left(D_6 | \overline{X}\right)\right] \tag{10.2}$$

The events are:

B_l: RS > LLR;
D_6: D_s > MAD;
X: Damage is not present at the location.

Example 10.1 shows potential orders of magnitude and illustrates how in the design process damage resistance becomes the starting point in target setting.

Example 10.1: This example works with a modification of Eq. (10.2) as follows,

$$P(\overline{U}) = P(\overline{X}) \cdot \left[\sum_{i=1}^{5} P\left(\overline{B}_l | \overline{X}_i D_i\right) \cdot P\left(D_i | \overline{X}_i\right) + P\left(D_6 | \overline{X}\right)\right] \tag{10.3}$$

The terms in the sum will be assessed first,

i	1st factor	2nd factor	Product	Total
1–2	0.889	10^{-6}	0	
3	0.001	0.1	10^{-4}	
4	0.01	0.01	10^{-4}	
5	0.1	0.001	10^{-4}	
				$3 \cdot 10^{-4}$

The total value of the Eq. (10.3),

$$P(\overline{U}) = 10^{-2} \cdot 4 \cdot 10^{-4} = 4 \cdot 10^{-6}$$

where $P(D_6|\overline{X})$ is a set equal to 10^{-4}. So both this term and the third column in the table represents damage resistance, while the second column is a representation of damage tolerance.

Safety also forces common accidental damage initiation to be "contained" to region 4 or below with a high probability for many reasons. Survival for a whole inspection period with lost structural integrity (damage size in region 5) is an unlikely event and such situation should be avoided. Another reason is illustrated in Section 10.2. It involves damage growth. The requirements and prudent practices necessitate that damage growth be considered for all environments and conditions. Beside the difficulty in proving a "negative" the mere practicality of considering "ALL," especially in the context of innovation, makes the uncertainty of growth such that a large range of growth situations, including process failures, must be considered and Example 10.3 shows one way of doing it.

10.1. ULTIMATE STATIC STRENGTH CRITICAL STRUCTURE

The design of ultimate strength critical composite structure is all by itself considerably more complicated than what the process is for more conventional aluminum structure. The notch-sensitivity of composites and the effects of impact damage have fostered practices and produced guidance materials that have promoted design criteria that include "open-hole" compression and "filled-hole" tension requirements in addition to accidental damages (preferably up to detectable levels) in critical locations, for both material allowables and element (including panel), allowables.

The probability of lost ultimate integrity could be expressed as,

$$P(\overline{U}_{\text{u}}) = P(\overline{U}_l) + P(\overline{U}_H) \qquad (10.4)$$

The first term involves accidental damage for ultimate and is,

$$P(\overline{U}_I) = P(\overline{B}_{\mathrm{u}}|D_2\overline{X}) \cdot P(D_2|\overline{X}) \cdot P(\overline{X}) \tag{10.5}$$

The second term expresses the loss of integrity in conjunction with fastener holes, (e.g. open-hole compression or filled-hole tension),

$$P(\overline{U}_H) = P(\overline{B}_{\mathrm{u}}|\mathrm{FOT}) \cdot P(\mathrm{F}|\mathrm{OT}) \cdot P(\mathrm{O}|\mathrm{T}) \cdot P(\mathrm{T}) \tag{10.6}$$

The following events are involved,

F: Fastener is installed;
O: Worst hole condition exists;
T: Worst temperature condition exists,

and worst moisture conditions prevail.

Example 10.2 shows orders of magnitude considered representative for safe objectives.

Example 10.2: We assume that a design fastener location is considered (an alternative could be a repair location). It is also assumed that damage *B*-value allowable values are used. Then the damage part could be written by using Eq. (10.5),

$$P(\overline{U}_I) = 10^{-1} \cdot 10^{-2} \cdot 10^{-2} = 0.1 \cdot 10^{-4}$$

and Eq. (10.6) would yield the other part of ultimate integrity,

$$P(\overline{U}_H) = 10^{-1} \cdot 1 \cdot 10^{-2} \cdot 10^{-1} = 10^{-4}$$

Totally, then the probability of loss of ultimate integrity is $= 1.1 \cdot 10^{-4}$, and a detailed investigation is required to determine criticality.

10.2. DAMAGE GROWTH AND DAMAGE RESISTANCE

The typical situation involves expected growth rates between "zero-growth" and a maximum that is measured here in a predetermined size increase in a certain number of inspection periods. Example 10.3 illustrates the concepts and some realistic expectations extracted from the composite damage database that includes effects of the "freeze–thaw" cycles encountered in normal flight.

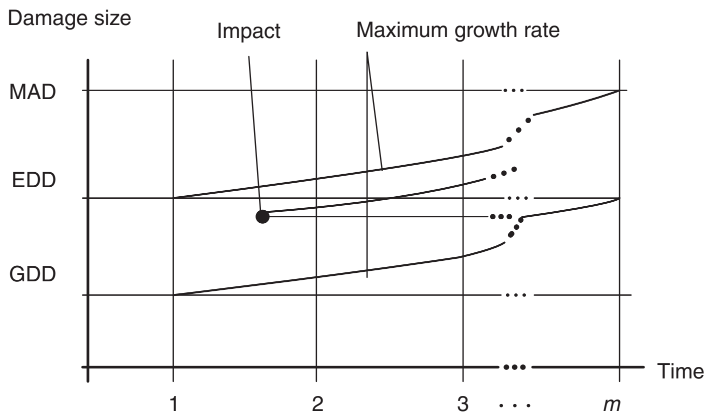


Figure 10.1. Damage growth.

Example 10.3: This example is based on the assumption that a maximum growth rate has been encountered, inferred or legislated as compatible with the inspection program and established damage resistance levels. The distribution between "no-growth" and the maximum growth is assumed uniform. Figure 10.1 illustrates a situation where an inspection period is n flights and the period of prescribed growth is m inspection intervals. We assume an exponential growth and GDD $= l$ and EDD $= L$, which yields,

$$D_5 = l\mathrm{e}^{(i/mn)\ln[(l+L)/l]} \tag{10.7}$$

where it is assumed that growth has added L to the size in m inspection intervals. The probability of having growth into region 5 in n flights is,

$$P(D_5|n) = \frac{l\left[\mathrm{e}^{(l/m)\ln[(l+L)/l]} - 1\right]}{L + l\left[\mathrm{e}^{(l/m)\ln[(l+L)/l]} - 1\right]} \tag{10.8}$$

Eq. (10.5) represents a uniform distribution based on ratios between the region 5 part and the total, and it can be rewritten as,

$$P_r = \frac{1}{1 + \frac{L}{l}\left[\mathrm{e}^{(l/m)\ln[(l+L)/l]} - 1\right]^{-1}}$$

With the following added assumption, $l = L$, and starting damage size in region 4, we get the following probabilities of growth to region 5 in n flights,

m	p_r
2	0.29
3	0.21
4	0.16
.	.
10	0.07

Furthermore, the probability that the damage at flight 1 in region 4 will grow to region 5 at the next inspection is,

m	p_{su}
2	$3 \cdot 10^{-4}$
3	$2 \cdot 10^{-4}$
4	$1.6 \cdot 10^{-4}$
$\rightarrow$.	10^{-4}
10	

The presumed starting damage size in region 4 illustrates how damage resistance and damage growth goals must be set together in order to produce rational design criteria. It is worth mentioning that it certainly is important to consider the detectability of damage in region 4 or the value of the choice of region 4 relative to the probability of detection.

10.3. DAMAGE TOLERANCE

A reasonable approach toward designing composite structure, whether damage tolerance critical or not, is to determine a criterion for desired damage tolerance in terms of a quality associated with one type of damage, e.g. "one failed stringer and the adjacent skin damaged to a width equal to the stringer spacing" (a traditional choice in the metal world). Then a significant part of the design work will be to make sure that all the realistic damage types that could be inflicted in service are less critical. Even when the structure is static strength critical, a considerable amount of the design work must be focused on structural behavior with damage present. Furthermore, safety requires an unconditional search for substantial, well-defined damage tolerance and a good, steadily improving understanding of the threat environment.

The new generations of composite materials and structures, even though much tougher and damage tolerant than its predecessors, also have made the design process more demanding, and the focus on explicit evaluations of safety levels more important.

An inevitable conclusion for design of composite structures is that,

"Damage resistance";
"Damage growth";
"Damage tolerance";
"Damage detection,"

all are essential cornerstones in safe innovation and the primary "drivers" in the design of composite structures.

A balanced design also needs a detailed definition of the threats, e.g. their mass, impact radius, speed and direction, so that relative importance from a damage tolerance standpoint can be determined. So, in conclusion, structural design requires a detailed consideration and balance between the mentioned "drivers." Something that is a lot more important for composites than metals, because of the much larger number of design variables.

A very productive approach to both ultimate static strength design and damage tolerance design is to categorize damage in size regions and to produce allowable data for both types of criticality.

10.4. DISCRETE SOURCE DAMAGE

Discrete source damage is a term used for specific accidental damage that cannot be inflicted without the pilot's knowledge. There are often two requirements, the primary structure must not be penetrated in some cases, and the residual strength always must be such that the structures can carry "get-home loads" (for wings, e.g. 70 per cent of limit load).

These kinds of damage are identified in the international regulations. The two major types are "Bird-Strikes" and "Fragment Strikes" from disintegrating engine turbines. Penetration is not allowed due to bird-strike in the front pressure bulkhead and structure surrounding the windshield (and of course the windshield), on the leading edge of the wing, the front spar must survive (fuel tank intact) and the same is true for the empennage even though a bigger bird (8 pounds) could be involved. The last few years, incidents involving hailstones in flights have been occurring (whether due to changes in operating procedures or not, is not clear), and it is obviously prudent design to consider realistic sources of damage even though not covered by regulations.

A number of events are part of loss of "Discrete Source Structural" integrity, $\overline{U}_{\mathrm{DS}}$, and Eq. (10.9) describes the different situations,

$$P(\overline{U}_{\mathrm{DS}}) = P(\overline{U}_{\mathrm{DS}}\overline{Z}) + P(\overline{U}_{\mathrm{DS}}Z) \tag{10.9}$$

The second term on the right-hand side represents "Degradation," and is for most of today's atmospheric environments negligible except for supersonic and faster flights.

The negligible degradation case can be written as,

$$P(\overline{U}_{\mathrm{DS}}) = P(\overline{U}_{\mathrm{DS}}P\overline{Z}) + P(\overline{U}_{\mathrm{DS}}\overline{P}\,\overline{Z}) \tag{10.10}$$

This equation can be expanded as,

$$P(\overline{U}_{\mathrm{DS}}) = P(\overline{U}_{\mathrm{DS}}|P\overline{Z}) \cdot P(P|\overline{Z}) \cdot P(\overline{Z}) + P(\overline{U}_{\mathrm{DS}}|\overline{P}\,\overline{Z}) \cdot P(\overline{P}|\overline{Z}) \cdot P(\overline{Z}) \tag{10.11}$$

The first factor in the first term of the right-hand side of Eq. (10.11) (and analogously the first factor in the second term), can be written as,

$$P(\overline{U}_{\mathrm{DS}}|P(\overline{Z})) = P\{[\overline{R} \cup (R\overline{B}_D)]|P\overline{Z}\} \tag{10.12}$$

The following basic events are involved,

$\overline{B}_D$: Residual strength is less than "Get-home" load requirements, RS $\leq$ GHR;
$\overline{G}$: Surprise event, not included in regulations;
P: Penetration;
R: Damage is less than discrete source value, $D_s \leq$ DSD;
R_1: $D_s \leq$ ODD (obvious damage);
R_2: ODD $\leq D_s \leq$ DSD (discrete source damage);
$\overline{U}_{\mathrm{DS}}$: Loss of discrete source integrity;
$\overline{Z}$: Discrete source event.

The purpose of G is to emphasize the need to assess emerging threats.

Eq. (10.12) can be expanded as,

$$\begin{aligned} P(\overline{U}_{\mathrm{DS}}|P\overline{Z}) = P(\overline{R}|P\overline{Z}) + P(\overline{B}_D|R_1P\overline{Z}) \cdot P(R_1|P\overline{Z}) \\ + P(\overline{B}_D|R_2P\overline{Z}) \cdot P(R_2|P\overline{Z}) \end{aligned} \tag{10.13}$$

The first factor of the second term and third term on the right-hand side relates to damage tolerance. The second factor involves damage resistance. It will be clear that an effective balance between damage resistance and damage tolerance is of the essence in the design of composite structure. Example 10.4 illustrates orders of magnitude.

Example 10.4: The probability that a certain exposed location of an airplane is subject to a discrete source event will be assumed to be based on the notion that an airplane could have experienced three events during its service life, 30 000 flights, and there are 100 exposed locations on the airplane. So the probability of a discrete event, $P(\overline{Z})$ can be written with the following participating events,

A_{DS}: The airplane is impacted;
E_{DS}: The PSE location is impacted,

as,

$$P(\overline{Z}) = P(A_{\mathrm{DS}}E_{\mathrm{DS}}) = P(A_{\mathrm{DS}}) \cdot P(E_{\mathrm{DS}}|A_{\mathrm{DS}}) = 10^{-4} \cdot 10^{-2}$$

We now assume that the event represents a bird-strike, then,

$$P(\overline{U}_{\mathrm{DS}}|P\overline{Z}) = 1$$

$$P(P|\overline{Z}) = 10^{-3}$$

$$P(\overline{Z}) = 10^{-6}$$

and Eq. (10.13) yields,

$$P(\overline{U}_{\mathrm{DS}}|P\overline{Z}) = 10^{-3} + 10^{-3} \cdot 1 + 10^{-1} \cdot 10^{-2} = 3 \cdot 10^{-3}$$

and the total,

$$P(\overline{U}_{\mathrm{DS}}) = 1 \cdot 10^{-3} \cdot 10^{-6} + 3 \cdot 10^{-3} \cdot 10^{-6} = 4 \cdot 10^{-9}$$

If we, however, deal with the impact of a turbine blade fragment, then,

$$P(\overline{U}_{\mathrm{DS}}|\overline{P}\,\overline{Z}) = 0$$

Eq. (10.13) yields,

$$P(\overline{U}_{\mathrm{DS}}|P\overline{Z}) = 10^{-3} + 10^{-3} \cdot 10^{-1} + 10^{-1} \cdot 10^{-2} \approx 2 \cdot 10^{-3}$$

The total probability of loss of "discrete source" integrity for turbine blade fragments becomes,

$$P(\overline{U}_{\mathrm{DS}}) = 10^{-3} \cdot 10^{-6} + 2 \cdot 10^{-3} \cdot 10^{-6} = 3 \cdot 10^{-9}$$

Clearly the relative criticalities between different threats depend on how damage tolerance and damage resistance are balanced in the design.

New threats are emerging, for example hailstone impact in flights. The example is not covered by the existing FAA regulations (or JAR). Recent events, causing severe structural damage, have been recorded, and prudent design, especially in the face of innovation, should consider including these well-established threats, even though it is arguable that "Safe Operation" would prevent them from materializing.

When new threats are being considered, deciding how to prevent penetration is a very important first step in design, and penetration should be prevented whenever practicable. Eq. (10.11) (with $\overline{G}$ replacing $\overline{Z}$) should guide the safety considerations in the design, and Discrete Source Damage criteria should be a substantial part of safe design.

10.5. DESIGN VARIABLES

Composite structures have the design variables, typically considered in metal design; thicknesses, areas, widths, heights, spacing, attachments, etc. However, it also incorporates a number of additional variables (even after material and concept selections have been made); fiber directions, material forms, lay-ups, processes, etc.

Figure 10.2 shows "Damage resistance" of a given structural concept. This type of information can then be used to select the damage resistance that is needed, so that combination with a growth situation, defined as in Figure 10.1, and a safe damage state can exist between inspections, especially in situations when the accidental damage is inflicted in the early part of an inspection interval.

Clearly laminate lay-ups and stringer shapes and spacing complicates the design data requirements, as does the selection between different concepts. The process is very similar, though, to what is being done for buckling allowable values, especially the panel testing part. There are some similarities in the presentation of "residual strength" data.

The use of "damage criteria" with damage more critical than the realistic threats produced in service, pays off dramatically in this part of the "design data generation process." Different damage types and locations would make "design space" too large for most cases. Figure 10.3 illustrates design data for a specific damage.

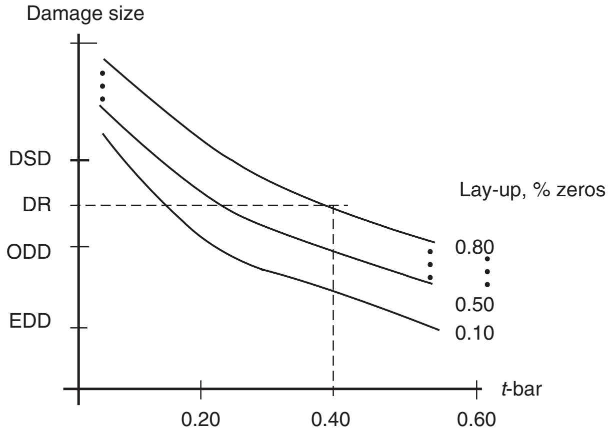


Figure 10.2. Damage resistance; damage size vs *t*-bar.

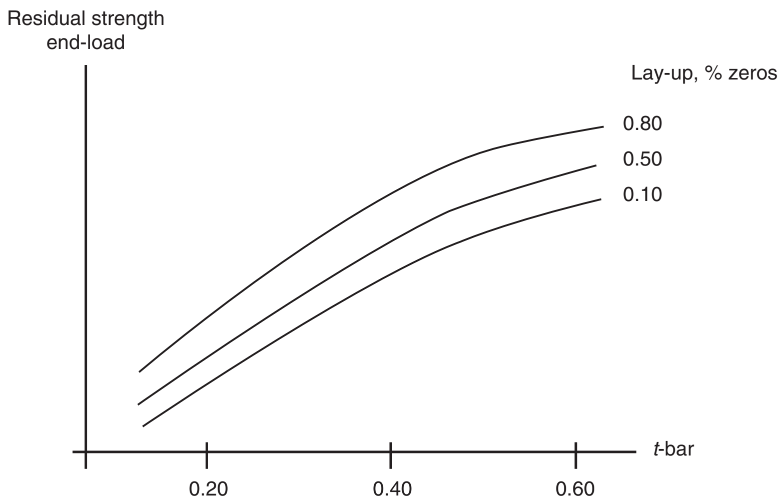


Figure 10.3. End-load capability for a specific damage and concept.

Figure 10.2 shows the complexity of the design situation. In addition to having fixed the structural concept and its characteristic variables, the figure applies to a specific definition of the lay-up, e.g. 50/25/25, where the latter two are implied.

There is also an implication of what probability value is used in the plot. It becomes clear from practicalities that the design process is best served by allowables-like information, and if one considers both damage size and residual strength, the following probability is important,

$$P(BD|L_u T_b) = P(B|DL_u T_b) \cdot P(D|L_u T_b) \qquad (10.14)$$

The events are:

B: RS > SR (strength requirement);
D: D_s < DR (design requirement);
L_u: Lay-up picked;
T_b: t-bar picked.

Example 10.5: This example illustrates the design approach used in Eq. (10.14). If Figure 10.2 represented a (99/95)-value, (analogous to A-value), we would have,

$$P(BD|L_u T_b) = 0.99 \cdot P(B|DL_u T_b)$$

So, whatever allowable-value quality used in Figure 10.3 would essentially be preserved. The importance of selecting a "Criteria Damage" for design is again illustrated. If as in previous examples, the order of magnitude of the residual strength probabilities is 90 per cent, then the challenge will lie in the demonstration that the "Criteria Damage" is more critical than what is inflicted by the practical service environment.

The damage growth rates, the detectability in major inspections and accessibility of "walk-around" inspections contribute to the complexity of any optimization, even when only weight is considered in the merit function. It seems that it is only when materials and concepts are given, that an optimization of weight for a damage tolerance critical structure is practical, as both damage resistance and damage growth need empirical sizing algorithms.

10.6. CRITERIA DAMAGE

Every PSE has potentially a number of different damage types that must be considered in its design. The inspection method of choice (determined in design as to required precision) is the baseline for the damage regions considered in the residual strength determination. Previous discussions have dealt with a case of six regions. We will continue exploring that case.

Ultimate strength is ordinarily based on a damage size that is "visible" or preferably a damage size and nature with good detectability. Situations where both external and internal damage sizes are important for detection are illustrated in Figure 10.5.

The external damage sizes can be considered as belonging to three different regions depending on internal damage size. They are:

D_{e1}: $0 < D_e <$ NDD;
D_{e2}: NDD $< D_e <$ EVD (easily visible damage);
D_{e3}: EVD $< D_e < D_s$.

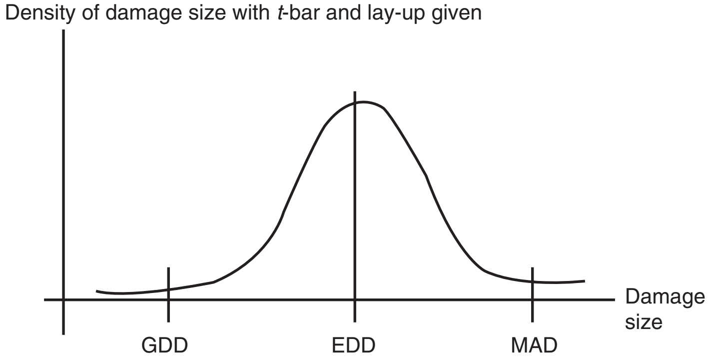


Figure 10.4. Probability density with t-bar and lay-up given.

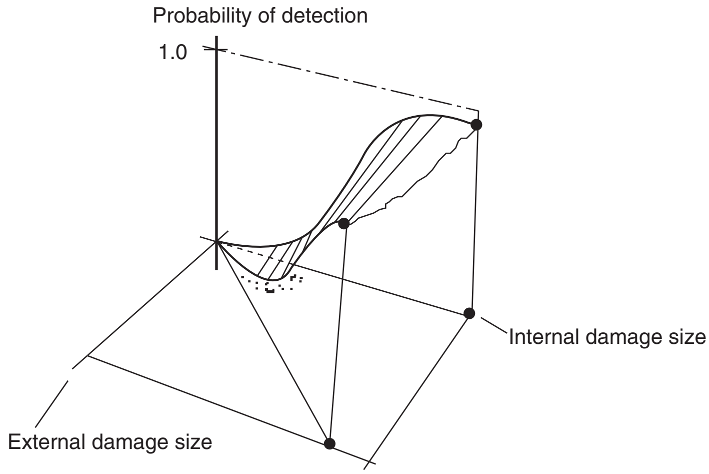


Figure 10.5. Probability of detection as a function of damage sizes.

Figure 10.5 shows among others, a case both possible and troublesome; zero external damage size with arbitrary internal damage size. One can imagine a damage originating during manufacturing and growing to threatening size in service. The potential that this type of event has a reasonable probability of occurring under the "right circumstances," raises questions about the definition of "ultimate damage." It also puts damage growth rate determinations and requirements under debate.

One common approach, to determining damage sizes to include in ultimate structural allowable values, is to invoke Barely Visible Damage, BVID definition based on "external damage." Obviously that is not practical. Figure 10.4 shows, for "naturally selected intervals," how detectability can lead to a rational design criterion for internal damage sizes. Figure 10.4 is an example of a design chart, which together with Figure 10.5, can be used to relate internal damage for ultimate and limit loads to external damage for different thicknesses and lay-ups (one chart for each set), if the inspection method requires external damage for reliable detection. A number of approaches could be considered:

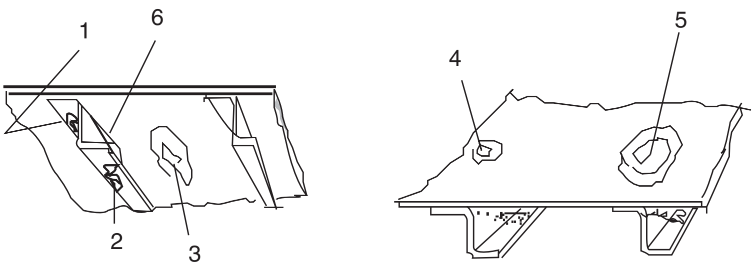
1. An inspection approach using internal damage could be used and the damage size GDD (see previous definition) could be used for the interval definition;
2. Damage growth rates aimed for in design could be such as to support detection before any significant growth has taken place;
3. Damage containments in detail design or structural concept selection that constrains damage growth to a prescribed size;
4. Change criteria for damage tolerance critical structure, so that a loss of ultimate strength is not treated as a safety issue, especially considering the fact that it must be lost in order to be a threat to damage tolerance, which in this case is the guardian of safety.

The general situation for a PSE is that there are a number of damage types to consider in the design. Figure 10.6 shows an example for composite skin–stringer construction.

These six types of damage, to consider, would all be taken care of by a criterion damage that was more critical than each of them. The following inequality would describe a desirable situation,

$$\mathrm{RS_C} \leq \mathrm{RS_1} \leq \cdots \leq \mathrm{RS}_n \tag{10.15}$$

where the first variable represents the criterion. The use of criteria damage for a PSE would reduce the required design, analysis and testing work, and allowables would only be required for one damage type for each PSE.



1. Broken fibers and cracked matrix = 2 = 4
3. Delamination = 5
6. Disbond

Figure 10.6. Damage types for composite skin–stringers.

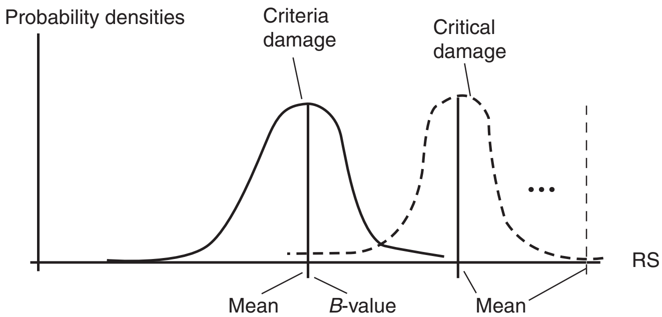


Figure 10.7. Residual strength density comparisons.

This would be what the traditional approach would have produced, and an investigation of the simplest way to produce allowable values based on mean value test data will be explored in Example 10.6.

Example 10.6: Eq. (10.15) will be used as the basis for criticality. Figure 10.7 illustrates a normally distributed set of random variables (residual strength).

We will now require that the mean of the "Criteria Damage" is also the *B*-value of the most critical potential service damage. We assume a coefficient of variation, $C_v = 0.1$.

$$t_i^{0.10} = \frac{\mu_C - \mu_i}{\sigma_i} \Rightarrow \Phi(t_i^{0.10}) = -1.30 = \frac{k\mu_i - \mu_i}{C_v} \Rightarrow k = 0.87$$

so if we, e.g. choose "a cracked stringer and a skin crack width adjusted" so that

$$\mu_C = 0.87 \cdot \mu_i \Rightarrow F_{RSC} = 0.87 \cdot F_{RSi}$$

we can make the case that we are using *B*-values, and the sizing would produce

$$\bar{t} = \frac{N^{\lim}}{F_{RSC}}$$

and both the allowables testing and the design work would be reduced substantially. A review of damage criticalities and alternative views can be found in Chapter 11. However, the question of critical damage in a specific location will be addressed later in the book.

10.7. CRITICAL DAMAGE TYPE

Each principal structural element has a number of locations that are critical to the design. Each location has a critical type of damage that constitutes the worst safety threat at that location. One way to rate criticality is to order damage types by probability of an unsafe state due to the damage at hand; with the largest probability of an unsafe state would indicate the most critical damage type.

A specific location is threatened by n damage types i, and the state involves both non-detection, $\overline{H}_1$, and loss of damage tolerance integrity. The probability of not detecting a specific damage can be expressed as,

$$\sum_{j=1}^{3} P\left(\overline{H}_1 T_i D_{ej} D_5 \overline{X}_l\right) = \sum P\left(\overline{H}_1 | D_5 D_{ej} T_i \overline{X}\right) \cdot P\left(D_5 D_{ej} T_i \overline{X}_l\right) \tag{10.16}$$

The participating sub-events are,

T_i: Damage type i;
D_5: EDD $\leq D_s \leq$ MAD;
D_{ej}: External damage size in range j;
$\overline{X}_l$: Damage is present;
$\overline{B}_1$: RS $\leq$ LLR at start of the first flight after major inspection;
$\overline{H}_1$: Damage not detected before first flight;
$D_{TS} = D_5 D_{ej} T_i \overline{X}_l$; $D_{HS} = D_5 T_i \overline{X}_l$, where D_{TS} is total damage state and D_{HS} is hidden damage state;

and Eq. (10.16) can be rewritten as

$$P(D_{TS}) = P(D_{TS} B_1) + P\left(D_{TS} \overline{B}_1\right) \Rightarrow P(D'_{TS}) = P(D_{TS} \overline{B}_1)$$

where only the sub-set involving lost integrity enters into the expression of an "unsafe state," and Eq. (10.16) becomes,

$$P\left(\overline{S}_i\right) = \sum_{j=1}^{3} P\left(\overline{H}_1 | D_{TS}\right) P\left(\overline{B}_1 | D_{TS}\right) P(D_{ej} | D_5 T_i \overline{X}_l) P\left(D_5 | T_i \overline{X}_l\right) P\left(T_i | \overline{X}_l\right) P\left(\overline{X}_l\right) \tag{10.17}$$

which can be written as,

$$P\left(\overline{S}_i\right) = P\left(D_5 | T_i \overline{X}_l\right) P\left(T_i | \overline{X}_l\right) P\left(\overline{X}_l\right) P\left(\overline{B}_1 | D_{TS}\right) \sum_{j=1}^{3} P\left(\overline{H}_1 | D_{ej} D_{HS}\right) P(D_{ej} | D_{HS}) \tag{10.18}$$

The next example will be used to study a specific set of damage types for a skin–stringer construction.

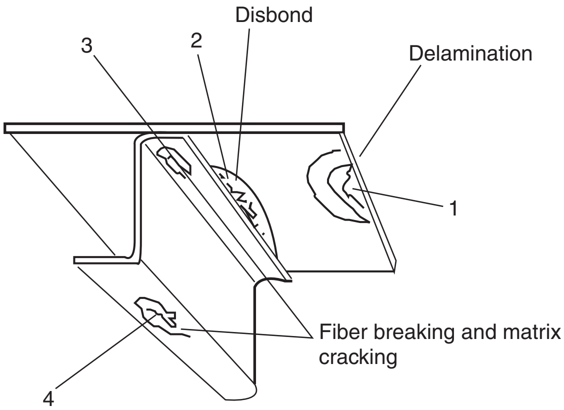


Figure 10.8. Damage types at a PSE location.

Example 10.7: We will assume that a "location" can be defined as the region considered in the creation of a skin–stringer allowable values and thicknesses (see Figure 10.8).

Four types of damage are being considered. The damage to the free flange is either inflicted in manufacturing or in the context of maintenance. The other three types are either caused by growth or by severe accidental damage or assumed accessible to "preflight inspection," and consequently inspected before every flight and detected safely, if safe operation is in place.

Representative expectations will be used to assess the different damage types and their specific probability of unsafe states.

We now assume that the focus is on a location which is not subject to accidental damage in service. Therefore we assume that only range 1 for external damage is under question.

The first type is characterized by,

$$P(\overline{S}_1) = 10^{-1} \cdot 10^{-2} \cdot 10^{-2} \cdot 10^{-1} \cdot P(\overline{H}_1|D_{\mathrm{TS}}) \cdot 1 = 10^{-6} \cdot P(\overline{H}_1|D_{\mathrm{TS}})$$

The second type has the following probability,

$$P(\overline{S}_2) = 10^{-2} \cdot 10^{-2} \cdot 10^{-2} \cdot 10^{-1} \cdot P(\overline{H}_1|D_{\mathrm{TS}}) \cdot 1 = 10^{-7} \cdot P(\overline{H}_1|D_{\mathrm{TS}})$$

The third type has,

$$P(\overline{S}_3) = 10^{-2} \cdot 10^{-2} \cdot 10^{-2} \cdot 10^{-1} \cdot P(\overline{H}_1|D_{\mathrm{TS}}) \cdot 1 = 10^{-7} \cdot P(\overline{H}_1|D_{\mathrm{TS}})$$

The fourth type of damage is represented by,

$$P(\overline{S}_4) = 10^{-4} \cdot 1 \cdot 10^{-2} \cdot 10^{-1} \cdot P(\overline{H}_1 | D_{\mathrm{TS}}) \cdot 1 = 10^{-7} \cdot P(\overline{H}_1 | D_{\mathrm{TS}})$$

One could choose to use different inspection methods for different types, or one could use more than one method or one approach at different "locations." That could make the last factor in each probability value the discriminating effect.

It is clear from the numbers that uncertainty is a big part of this assessment and that emerging service experience must be evaluated and tested against the a priori values. Especially when the design is based on "criteria damage" and service experience with the material, the process or the structural concept, it is very important that design criteria are robust enough, so that safety can be maintained for innovation even though there has to be processes in place that make it possible to absorb new insights into an existing new design without preserving questionable safety levels.

The whole purpose behind the relative criticality rating is to find a reasonable design criterion that can be exercised without expensive or superfluous testing and possibly be concentrated around mean value determination and *B*-value quality design data.

This requires a flexible definition of the "criteria damage" so that the mean strength can be adjusted through the damage detail definition so that the mean of the criteria damage residual strength, μ_{C}, compares to the critical damage mean residual strength, μ_i as the following requirements state,

$\Pr(\mu_{\mathrm{C}} < 0.87 \cdot \mu_i) < 0.10$, and *B*-value requirements would be satisfied up to a $C_{\mathrm{v}} = 0.1$

Composite structural design, whether critical for damage tolerance or not, requires that the safety aspects of integrity with structural damage is recognized as a primary design concern.

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Chapter 11
Damage and Detection

Damage present at the first flight after a major inspection can constitute a significant safety problem. The situation has been recognized, in the discussion of risk management, as the main element in establishing an acceptable safety level for flight vehicles.

There are a few situations that need further scrutiny. Some of the more severe are:

1. Damage initiated in the manufacturing process has insignificant external damage size indicators and grows to ''region 5 internal damage during service'';
2. Severe damage (region 5) initiated after major inspection but before start of the first flight and not accessible to ''preflight walk-around inspections'';
3. Unreported and undetected major damage resulting from ground accidents for cases when external damage size is disproportionately small compared to internal damage size;
4. Accidental in-service damage with ''faint'' and fading external damage indicators;
5. Significant strength reduction due to processing failures or in-service degradation;
6. Selection of maximum damage size for ultimate static strength requirements.

Damage, detection and inspection approaches are important ingredients in design to avoid ''Unsafe States'' in composites. A detailed investigation of the probability of an unsafe state for a specific PSE is conducted in the next section.

11.1. FAILED DETECTION

Inspection methods selected for service can depend on both internal and external damage sizes for detection. ''Visual'' inspection would depend on external damage size for detection. The ''tap-test'' would be expected to depend on a combination of the two. And inspections based on acoustic response would be dominated by internal damage size. The critical situation would deal with loss of structural integrity with the unacceptable integrity remaining between the inspection at T and the first flight 1 (after inspection).

An unsafe state at PSE k, $\overline{S}_k$ is represented by n locations j, $\overline{S}_{kj}$.

The probability of an unsafe state is,

$$P(\overline{S}_k) = P(\overline{S}_{k1} \cup \overline{S}_{k2} \cup \cdots \cup \overline{S}_{kn}) = \sum_{j=1}^{n} P(\overline{S}_{kj}) \tag{11.1}$$

We will add the definitions of the following events to the already established set,

D_{e1}: $0 \leq D_e \leq \text{NDD}$ where, D_e is external damage size;
D_{e2}: $\text{NDD} \leq D_e \leq \text{EVD}$ (easily visible damage);
D_{e3}: $\text{EVD} \leq D_e \leq D_s$;
$\overline{X}_T$: Damage present at T;
$\overline{X}_1$: Damage present at 1;
$\overline{X}_{T1}$: Damage present at both T and 1.

With three regions for external damage, one of the terms in Eq. (11.1) can be written as,

$$\begin{aligned} P(\overline{S}_{kj}) = \sum_{i=1}^{3} & P(\overline{B}_1 | D_{51} \overline{X}_{T1}) \cdot P(D_{51} | D_{5T} D_{eiT} \overline{X}_{T1}) \\ & \cdot P(D_{5T} | D_{eiT} \overline{X}_{T1}) \cdot P(D_{eiT} | \overline{X}_{T1}) \cdot P(\overline{X}_{T1}) \end{aligned} \tag{11.2}$$

This equation can be rewritten, and if we accept the three ranges of external damage and the formulation of safety presented in Chapter 1 the result is,

$$\begin{aligned} P(\overline{S}_{kj}) = \sum_{i=1}^{3} & P(\overline{B}_1 | D_{51} D_{eiT} \overline{X}_{T1}) \cdot P(\overline{H}_T | D_{5T} D_{eiT} \overline{X}_{T1}) \\ & \cdot P(D_{5T} | D_{eiT} \overline{X}_{T1}) \cdot P(D_{eiT} | \overline{X}_{T1}) \cdot P(\overline{H}_T) \end{aligned} \tag{11.3}$$

Here the first factor represents the probability of acceptable residual strength, (RS < LLR in region 5). The second factor describes the probability of "non-detection" under prescribed damage size regions. The third factor gives the probability of a region 5 internal damage size, given an external damage size in range i. The fourth factor deals with the probability of an external damage size in range i, given that damage is present. Finally, the fifth factor is the marginal probability of "non-detection."

A number of examples will now be investigated to illustrate the effects of internal and external damage and consequences on detection. However, first we need to explore two details. One, an alternative formulation of Eq. (11.3), which can be rewritten as,

$$P(\overline{S}_{kj}) = P(\overline{B}_1|D_{51}\overline{X}_1) \cdot \sum_{i=1}^{3} P(\overline{H}_T|D_{5T}D_{eiT}\overline{X}_{T1}) \cdot P(D_{5T}|D_{eiT}\overline{X}_{T1}) \cdot P(D_{eiT}|\overline{X}_{T1}) \cdot P(\overline{H}_T) \tag{11.4}$$

where the expansion took place after the first step in the chain-rule expansion of the total event. Two, the background of the last factor in both Eqs. (11.3) and (11.4). Suppose that the background behind this factor includes the probability of a severe impact at a just repaired site, then the following event is pertinent,

$$p_{2d} = \sum P(D_{5T}D_{eiT}\overline{X}_T H_T R_T Y_{T1} D_{51}\overline{H}_{T1}) = \sum_{i=1}^{3} P(\overline{H}_{T1}|D_{5T}D_{eiT}\overline{X}_T R_T Y_{T1} D_{51} H_T) \cdot P(D_{51}|D_{eiT}D_{5T}\overline{X}_T H_T R_T Y_{T1}) \cdot P(Y_{T1}|D_{eiT}D_{5T}\overline{X}_T R_T H_T) \cdot P(R_T|D_{5T}D_{eiT}\overline{X}_T H_T) \cdot P(H_T|D_{5T}D_{eiT}\overline{X}_T) \cdot P(D_{5T}|D_{eit}\overline{X}_T) \cdot P(D_{eiT}|\overline{X}_T) \cdot P(\overline{X}_T) \tag{11.5}$$

This is a complicated event because it consists of so many sub-events; a severe damage present at time T, damage detected, damage repaired, new severe accidental damage inflicted between the inspection and the first flight after inspection, and damage is not detected. The main reason for investigating this complex event is to discard it as insignificant in the evaluation of the last factor in Eq. (11.4). Eq. (11.5) can be simplified,

$$P_{2d} = \sum_{i=1}^{3} P(\overline{H}_{T1}|\overline{X}_T H_T R_T Y_{T1} D_{51}) \cdot P(D_{51}|\overline{X}_T D_{5T} R_T Y_{T1}) \cdot P(Y_{T1}|\overline{X}_T D_{5T} R_T) \cdot 1 \cdot P(H_T|D_{5T}D_{eiT}\overline{X}_T) \cdot P(D_{5T}|D_{eiT}\overline{X}_T) \cdot P(D_{eiT}|\overline{X}_T) \cdot P(\overline{X}_T) \tag{11.6}$$

The next example will demonstrate a range for orders of magnitude for the probability of this event, P_{2d}.

Example 11.1: This example deals with the events of Eq. (11.6). Values dependent on i will be listed within parenthesis in order $i = 1, 2, 3$.

$$p_{2d} = 10^{-3} \cdot 10^{-3} \cdot 10^{-3} \cdot 1 \cdot (10^{-1}, 0.5, 1.0) \cdot (10^{-3}, 10^{-2}, 10^{-1}) \cdot (10^{-1}, 10^{-2}, 10^{-3}) \cdot 10^{-2} \approx 1.5 \cdot 10^{-15}$$

Which illustrates the nature of the probability of this multiple damage case, and establishes in the comparison with the order of magnitude of the factor

$$P(\overline{H}_T) \approx 10^{-2}$$

a practically insignificant contribution.

We now can return to the probability of an unsafe state of a specific PSE at a specific location and consider the case when only internal damage is of importance for detection. The basic definition of the probability of an "unsafe state" then becomes,

$$\begin{aligned} P(\overline{B}_1 D_{51} D_{5T} \overline{X}_{T1}) &= P(\overline{B}_1 | D_{51} \overline{X}_{T1}) \cdot P(D_{51} | D_{5T} \overline{X}_{T1}) \cdot P(D_{5T} | \overline{X}_{T1}) \cdot P(\overline{X}_{T1}) \\ &= P(\overline{B}_1 | D_{51} \overline{X}_{T1}) \cdot P(\overline{H}_T | D_{5T} \overline{X}_{T1}) \cdot P(D_{5T} | \overline{X}_{T1}) \cdot P(\overline{H}_T) \end{aligned} \quad (11.7)$$

So the probability of an unsafe state at location j of PSE k is then described by Eq. (11.7), when only internal damage enters into detection. The next Example 11.2 deals with that situation.

Example 11.2: Maintaining the regime of values used in the previous examples, the probability of an unsafe state can be estimated in the following way:

$$P(\overline{S}_{kj}) = 10^{-1} \cdot 10^{-3} \cdot 10^{-3} \cdot 10^{-2} = 10^{-9}$$

where B-values are presumed in the first factor. For the case with five locations of about the same criticality, the probability of an unsafe state of PSE k becomes,

$$P(\overline{S}_k) \approx 5 \cdot 10^{-9}$$

The order of magnitude of the total gives an indication of what range is expected for the participating sub-probabilities to satisfy an overall airplane safety level of "one unsafe flight in 10^5." Allowable residual strength value probabilities, probability of non-detection for major internal damage, probability of damage resistance to large damage and the probability value for the damage tolerance rating (DTR), all can be dealt with in Eq. (11.7).

11.2. MANUFACTURING DAMAGE

Eq. (11.4) contains considerations for both external and internal damage, but in order to study the situation where detection is solely dependent on external damage we rewrite the guiding equation. The probability of an unsafe state of PSE k at location j can be expressed as,

$$P(\overline{S}_{kj}) = P(\overline{B}_1|D_{51}\overline{X}_1) \cdot \sum_{i=1}^{3} P(\overline{H}_T|D_{eiT}\overline{X}_{T1}) \cdot P(D_{5T}|D_{eiT}\overline{X}_{T1}) \cdot P(D_{eiT}|\overline{X}_{T1}) \cdot P(\overline{H}_T) \tag{11.8}$$

Eq. (11.8) is used as the guiding equation for Example 11.3, which displays the effects of an inspection method that only uses external damage for detection in service.

Example 11.3: This example illustrates the potential probabilities of sub-events in order to satisfy a vehicle requirement of "one unsafe flight in 100 000." We assume that B-value residual strength probabilities are used.

$$P(\overline{S}_{kj}) \approx 10^{-1} \cdot (1, 10^{-2}, 10^{-3}) \cdot (10^{-3}, 10^{-2}, 1) \cdot (0.9, 10^{-1}, 10^{-2}) \cdot 10^{-2} \approx 3 \cdot 10^{-6}$$

Quite clearly, one must evaluate this type of inspection procedure very carefully before it could support the levels of safety we are looking for.

It appears that the situation 1 described in the list, mentioned in the beginning of this Chapter, only can be encountered safely, if one uses an inspection method that does not rely solely on external sizes. The popular "approach" of "No-growth" is not "in the running," as time limits do not apply to the situation (the results of flaw growth in turbine blades during the sixties and seventies come to mind). So in conclusion, a proper choice of inspection approaches can assure a safe resolution of situation 1.

11.3. MAINTENANCE DAMAGE

In order to proceed to the second item on the list mentioned in the beginning of this chapter, we need to revisit the definition of "Unsafe State" and identify some related concepts. An "Undesirable State of Damage" is an important manifestation of a safety threat. We have chosen to deal with six damage regions, and we will continue to deal with that approach, although many alternatives exist and many variations are possible.

We define an "Undesirable Damage State," D_u, as,

$$D_u\text{: } D_s \in D_4 \cup D_5 \cup D_6$$

and "undesirable Residual Strength" $\overline{B}_1$ is unchanged.

The probability of a bad damage state can be written as,

$$P(D_u\overline{H}\,\overline{X}_1) = \sum_{j=4}^{6} P(\overline{H}|D_j\overline{X}_1)P(D_j\overline{X}_1)$$
$$= \sum_{j=4}^{6}\left[P(\overline{H}|D_j\overline{X}_1)\right]\cdot\left[P(B_1D_j\overline{X}_1) + P(\overline{B}_1D_j\overline{X}_1)\right]$$

An "unsafe state" can then be defined as involving loss of residual strength integrity due to an accidental damage between T and 1.

$$P(\overline{S}) = \sum_{j=4}^{6} P(\overline{H}_1|D_jY_{T1})\cdot P(\overline{B}_1|D_jY_{T1})\cdot P(D_j|Y_{T1})\cdot P(Y_{T1}) = P(Y_{T1})$$
$$\cdot\sum_{j=4}^{6}\left[P(\overline{B}_1|D_jY_{T1})\cdot P(D_j|Y_{T1})\cdot\sum_{i=1}^{3}P(\overline{H}_1|D_{ei}D_jY_{T1})\cdot P(D_{ei}|D_jY_{T1})\right] \tag{11.9}$$

This situation is focused on accidental damage during the time period major inspection to first flight after inspection. Eq. (11.9) will be used to study the details.

Example 11.4: The type of accidental damage required to cause loss of limit load integrity under these circumstances would be characterized by $i = 2, 3$ resulting in,

$$P(\overline{S}_1) = P(Y_{T1})\cdot\{10^{-2}\cdot 10^{-2}\cdot[10^{-2}\cdot 10^{-2} + 10^{-3}\cdot 10^{-1}]$$
$$+ 10^{-1}\cdot 10^{-3}[10^{-2}\cdot 10^{-1} + 10^{-3}\cdot 0.5] + 1\cdot 10^{-4}\cdot[10^{-2}\cdot 0 + 10^{-3}\cdot 1]\}$$
$$= P(Y_{T1})\cdot(2\cdot 10^{-8} + 1.5\cdot 10^{-7} + 10^{-7}) \approx 2.7\cdot 10^{-7}\cdot P(Y_{T1})$$

where the following values have been used:

$P(\overline{B}_1|D_4Y_{T1}) = 10^{-2}$; $P(\overline{B}_1|D_6Y_{T1}) = 1$;
$P(D_4|Y_{T1}) = 10^{-2}$; $P(D_6|Y_{T1}) = 10^{-4}$;
$P(\overline{H}_1|D_{e2}D_4Y_{T1}) = 10^{-2}$; $P(\overline{H}_1|D_{e2}D_6Y_{T1}) = 10^{-2}$;
$P(D_{e2}|D_4Y_{T1}) = 10^{-2}$; $P(D_{e2}|D_6Y_{T1}) = 0$;
$P(\overline{H}_1|D_{e3}D_4Y_{T1}) = 10^{-3}$; $P(\overline{H}_1|D_{e3}D_6Y_{T1}) = 10^{-3}$;
$P(D_{e3}|D_4Y_{T1}) = 10^{-1}$; $P(D_{e3}|D_6Y_{T1}) = 1$.
$P(\overline{B}_1|D_5Y_{T1}) = 10^{-1}$;
$P(D_5|Y_{T1}) = 10^{-3}$;
$P(\overline{H}_1|D_{e2}D_5Y_{T1}) = 10^{-2}$;
$P(D_{e2}|D_5Y_{T1}) = 10^{-1}$;
$P(\overline{H}_1|D_{e3}D_5Y_{T1}) = 10^{-3}$;
$P(D_{e3}|D_5Y_{T1}) = 0.5$

These values are judged reasonable for illustrating orders of magnitude for the sub-events in light of the requirements for a part of maintenance.

$$P(\overline{S}_1) \approx 2.7 \cdot 10^{-7} \cdot P(Y_{T1})$$

It appears that a reasonable value of the last factor above is,

$$P(Y_{T1}) \sim 10^{-2}$$

And it would make a contribution, similar designs have made in our previous examples. This example has demonstrated relative little influence from damage region 4. And damage region 6 is such that only the largest external damage range is important.

It must be kept in mind that the definitions of the regions and ranges must be created with the different contributions to safety in mind.

Two very important factors in this context are "the final inspection before the vehicle is brought back into service" and the thoroughness by which it is conducted and the discipline at "site" that makes the probability of damage inflicted in maintenance small

$$P(Y_{T1}) < 10^{-2}$$

in this case, and subject to an evaluation of specifics from case-to-case. The monitoring of uncertainties of damage sizes and detection–non-detection in service is a very important part of risk management.

11.4. ACCIDENTAL DAMAGE

The third item of the list, mentioned in the beginning of this chapter, deals with PSEs that are exposed to damage in service. The exposure makes the potential damage site accessible to preflight "walk-around" inspections. Therefore, the question of survival until detection is a very important safety consideration.

We will start with the survival of the first flight after damage has been inflicted. The probability of completing the flight (if detection implies repair), is,

$$p_c = p_d + \overline{p}_d \cdot p_s$$

where

p_d = Probability of detection;
$\overline{p}_d = 1 - p_d$;
p_s = Probability of surviving an arbitrary flight.

The probability of not detecting the damage on n flights $\bar{p}_{dn}$ is,

$$\bar{p}_{dn} = \bar{p}_d^n \tag{11.10}$$

and the probability of surviving n flights

$$p_{sn} = \left(p_d + \bar{p}_d\, p_s\right)^n \tag{11.11}$$

These situations will be investigated in Example 11.5.

Example 11.5: We will start by looking at Eq. (11.10), and a range of values will be considered for $n = 10$

$1-p_d$	$\bar{p}_d^n$
0.1	10^{-10}
0.3	$6 \cdot 10^{-6}$
0.5	10^{-3}

The probability of surviving n flights p_{sn} will now be evaluated for 10 flights,

p_d	$1-p_d$	p_s	$\bar{p}_d p_s$	$p_d + \bar{p}_d p_s$	p_{sn}	Comments
0.9	0.1	0.5	0.05	0.95	0.60	Suggested
0.7	0.3	0.5	0.15	0.85	0.20	Minima
0.9	0.1	0.7	0.07	0.97	0.74	←
0.9	0.1	0.9	0.09	0.99	0.90	←
0.7	0.3	0.9	0.27	0.97	0.74	←
0.5	0.5	0.9	0.45	0.95	0.60	

The suggested minima indicates that a reasonable safe level could be established with, e.g. a survival probability of,

$$p_s > 0.7$$

if one in addition, for large damage, would establish a quality level of $p_d > 0.99$. For this large damage, we would have,

$$p_{sn} > 0.97$$

and a reasonable survival probability would have been established.

The accidental damage of large size can be included under the safety umbrella, if high quality "walk-around" inspections routinely are performed, or if a reasonable probability of survival with lost integrity is assured either by the damage resistance "designed in" or by establishing damage region definitions that assure damage tolerance for large damage sizes.

11.5. PROCESS FAILURE, DEGRADATION AND DAMAGE

We will deal with failures in processing (unnoticed violations of the process specifications) that cause unacceptable reduction in material and structural properties. For damage tolerance critical structure, we are especially interested in reductions that affect residual strength either directly or indirectly. The regions we are specifically interested in are:

D_{r1}: $F_{\text{ult}} \geq F_{\text{RS}} \geq \dfrac{F_{\text{ult}}}{1+\text{MS}}$;

D_{r2}: $\dfrac{F_{\text{ult}}}{1+\text{MS}} \geq F_{\text{RS}} \geq 1.5 \cdot F_{\text{lim}}$;

D_{r3}: $F_{\text{lim}} \geq F_{\text{RS}}$.

Here the boundaries represent established allowable values. The expression:

$$f_u^{\max} = \frac{F_{\text{ult}}}{1+\text{MS}}$$

describes the maximum "applied" ultimate stress. F_{lim} is the limit for damage tolerance integrity, and F_{RS} is the degraded failure stress. So the first region defines excess ultimate strength (damage tolerance critical structure). However, if we were to change requirements so that all load cases would apply to both ultimate and damage tolerance (limit) requirements, the second region would disappear, and one could change damage requirements for ultimate, so the damage size which satisfies that requirement could be used

$$\therefore\ F_{\text{ult}} = 1.5 \cdot F_{\text{lim}}$$

and simpler world would follow where,

D_{r1}: $F_{\text{ult}} \geq F_{\text{RS}} \geq F_{\text{lim}}$;

D_{r2}: $F_{\text{lim}} \geq F_{\text{RS}}$.

The region D_{r1} represents "lost ultimate integrity," and D_{r2} "lost limit integrity." The probability of loss of integrity could then be written as,

$$P(\overline{U}) = P(\overline{U}_{\text{U}} \cup \overline{U}_{\text{L}}) = P(\overline{U}_{\text{U}}) + P(\overline{U}_{\text{L}}) - P(\overline{U}_{\text{U}}\overline{U}_{\text{L}}) = P(\overline{U}_{\text{U}}) \tag{11.12}$$

Eq. (11.12) could be rewritten as

$$P(\overline{U}) = P(D_{r1}) + P(D_{r2}) \tag{11.13}$$

which all by itself is no surprise, but it allows you to approach the safety argument in parts. The second term in Eq. (11.13) represents a "loss of damage tolerance integrity," which is unacceptable. Consistent with the orders of magnitude in the series of examples studied, we would require,

$$P(D_{r2}\overline{H}_{QC}) \le 10^{-9}$$

the left-hand side can be expressed as,

$$P(D_{r2}\overline{H}_{QC}) = \sum_{i=1}^{n} P(D_{r2i}\overline{H}_{QC}) \tag{11.14}$$

where

$$D_{r2} = D_{r21} \cup D_{r22} \cup \cdots \cup D_{r2n}$$

and Eq. (11.14) can be expressed as,

$$P(\overline{U}_1) = \sum P(\overline{H}_{QC}|D_{r2i})P(D_{r2i}) \tag{11.15}$$

This equation is studied in Example 11.6.

Example 11.6: We assume that $n = 5$, and that each term in Eq. (11.15) is equal, so that we can write,

$$P(\overline{U}_1) = n \cdot P(\overline{H}_{QC}|D_{r2i}) \cdot P(D_{r2i}) \le 10^{-9}$$

which yields

$$P(\overline{H}_{QC}|D_{r2i}) \cdot P(D_{r2i}) \le 0.2 \cdot 10^{-9}$$

The first factor can be considered a measure of quality control and the second process control. Considering that this deals with a severe set of cases, we would start by requiring,

$$P(D_{r2i}) \approx\approx 10^{-6}$$

resulting in,

$$P(\overline{H}_{QC}|D_{r2i}) \leq 0.2 \cdot 10^{-3}$$

This is an indication of needs and it is evident from this example that great care must be taken in establishing requirements for both processes. It is clear that this must be dealt with on case-by-case basis, as requirements are driven by the number of processes involved and the interaction between all four aspects (see Chapter 1) of safety.

The first term on the right-hand side of Eq. (11.13) is an important part of safe manufacturing, because it involves the bulk of all mistakes. In a manner analogous to above, we have,

$$P(D_{r1}\overline{H}_{QC}) = \sum_{i=1}^{n} P(D_{r1i}\overline{H}_{QC}\overline{G}) = \sum_{i=1}^{n} P(\overline{H}_{QC}|D_{r1i}\overline{G})P(D_{r1i}|\overline{G})P(\overline{G}) \quad (11.16)$$

where $\overline{G}$ is the material property reduction happened. Example 11.7 contains a detailed look of Eq. (11.16).

Example 11.7: International and Federal regulations require that ultimate static strength is maintained through the life of flight critical structures. Not because the violations themselves are safety losses, but because it compromises both Fail-Safety and modern Damage Tolerance integrities (limit load strength with damage). For locations with positive margin of safety, we assume the summation takes place for $i = 2, 3, \ldots, n$, and we assume that process failures in general are maintained at

$$P(\overline{G}) \approx 10^{-4}$$

So if we use the same requirement as in Example 11.6, we can write,

$$\sum_{i=2}^{n} P(\overline{H}_{QC}|D_{1ri}\overline{G})P(D_{1ri}|\overline{G}) \leq 10^{-5}$$

Now we assume $n = 5$ and that $i = 2$ is the most probable result and gradually reduced probability for increasing i-values,

$i = \ldots$	1st factor	2nd factor	Product	Total
2	10^{-4}	10^{-2}	10^{-6}	
3	10^{-4}	$0.5 \cdot 10^{-2}$	$0.5 \cdot 10^{-6}$	
4	10^{-5}	10^{-3}	10^{-8}	
5	10^{-5}	10^{-4}	10^{-9}	► $\sim 1.5 \cdot 10^{-6}$

The result for this order of magnitude of requirements for quality control and process control then becomes,

$$P(D_{r1}) = 1.5 \cdot 10^{-10}$$

The total from Examples 11.6 and 11.7 is,

$$P(\overline{U}_1) = 0.2 \cdot 10^{-9} + 0.15 \cdot 10^{-9} = 0.35 \cdot 10^{-9}$$

These two examples illustrate the importance of setting the process requirements in concert with the other safety requirements for the flight vehicle and to do the case-by-case evaluation.

This is an insidious type of damage, if not detected at the source. Depending on severity, it can be a totally unsafe situation leading to serious mishaps; the classical case of "kissing bonds" in adhesive (or matrix) bonds belong to this category.

11.6. IN-SERVICE DEGRADATION AND DAMAGE ("AGING")

The most challenging aspect of degradation is that available inspection methods will not detect it. So, e.g. can loss of ultimate strength occur without detection, which makes the structure in question a candidate for "Safe-life design."

International and Federal regulations require "Loss of Ultimate Strength," due to undetectable damage sizes, to be rendered safe by using a safety factor on life (not less than 3.0). Figure 11.1 and Example 11.8 illustrate orders of magnitude. Here the three curves can be looked at as three different environments and Example 11.8 shows some of the details.

Example 11.8: This example is based on an exponential degradation. It is assumed that ultimate strength will be based on degradation after three lifetimes. The following function for degradation will be used,

$$\mu = \mu_0 e^{-\lambda(t/T)^2} \tag{11.17}$$

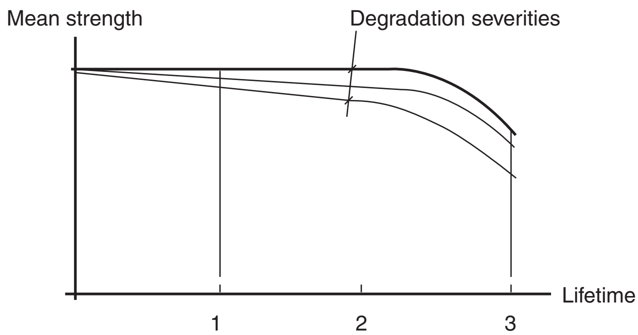

Figure 11.1. Alternative degradations in time.

We now assume that,

$$t = T \Rightarrow \mu = 0.67\mu_0 \Rightarrow \lambda_1 = 0.40 \quad \text{and } \sigma \text{ is unchanged}$$

If we assume a normal distribution Φ, we have,

$$\Phi(t) \text{ for a } B\text{-value is } 0.10 \Rightarrow t = -1.3 = \frac{(B/\mu_0) - 0.67}{C_v}$$

$$\text{and if we have } C_v = 0.10 \quad \text{then } F_{ult} = 0.54\,\mu_0$$

$$\text{if on the other hand, } \mu = 0.67\mu_0 \text{ and } \sigma \text{ is double, then}$$

$$-1.3 = \frac{(B/\mu_0) - 0.67}{2C_v} \Rightarrow F_{ult} = 0.41\mu_0$$

It is clear that weight penalties associated with safe-life safety factors are severe and the designer should either protect the structure from this kind of environment (design to avoid) or they should review the material selection and make a new material choice with degradation of the order of magnitude 10 per cent or less,

$$-1.3 = \frac{(B/\mu_0) - 0.9}{0.10} \Rightarrow F_{ult} = 0.77\mu_0$$

which compares to $0.87\mu_0$ for the pristine case.

And the design for degradation is very much a material choice and when one adds the fact that the reduction in strain energy release rates quite often are larger than what is the case for the strength values, it becomes clear that whenever possible, materials with degrading properties in the service environment should be avoided; as both damage resistance and damage tolerance properties reduction contribute to a decaying safety level.

11.7. GROWTH AND DAMAGE

Growth of damage inflicted during manufacturing or accidental damage inflicted in service can be a difficult phenomenon to come to grips with and often requires an inspection method that is sensitive to internal damage. The solution of choice often becomes the control of maximum growth by material choice, protection or control of operating strains, so that damage is detected before it has become unsafe. Example 11.9 investigates one situation of growth.

Example 11.9: This example deals with a maximum growth that is exponential. The growth is controlled to be moderate for the first three inspection intervals after infliction. Figure 11.2 describes the details and illustrates the assumption that the damage sizes due to growth are uniformly distributed between the consequences of "no-growth" to maximum growth (exponential).

The example deals with the situation of region 3 damage sizes. It controls the growth to L, during three inspection intervals. The growth is assumed to be exponential and expressed for the maximum as,

$$D_s = \mathrm{GDD} \cdot e^{0.333 \ln (\mathrm{EDD}/\mathrm{GDD})}$$

We now assume that,

$$\frac{\mathrm{EDD}}{\mathrm{GDD}} = \frac{L + \mathrm{GDD}}{\mathrm{GDD}} = 2 \Rightarrow D_s = \mathrm{GDD} \cdot 2^{(n/3)} \tag{11.18}$$

The uniform distribution is used to determine the participating probabilities of damage size. The probability of an unsafe state at the end of the fourth

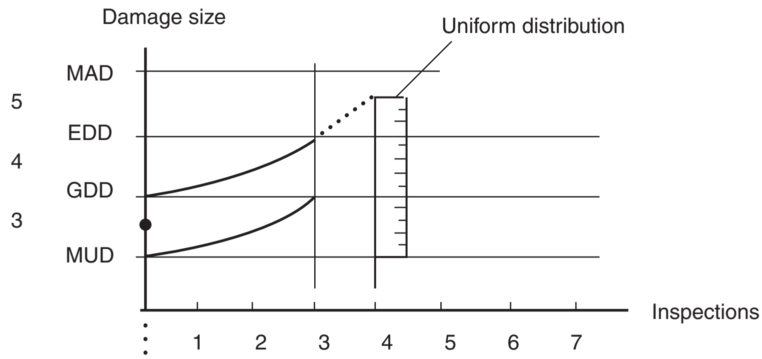

Figure 11.2. Growth and damage size distribution.

inspection interval is

$$P(\overline{S}_4) = P(\overline{B}_4 D_{54}) \cdot P(\overline{H}_3 D_{43}) \cdot P(\overline{H}_{42} D_{42}) \cdot P(\overline{H}_1 D_{41}) \cdot P(\overline{H}_0 D_{30}) \qquad (11.19)$$

where each factor

$$P(\overline{H}_j D_{ij}) = P(\overline{H}_j | D_{ij}) \cdot P(D_{ij})$$

is estimated by this equation.

The probabilities $P(D_{ij})$ are now determined,

$$\begin{aligned}
n &= 1 \quad D_s = \text{GDD} \cdot 1.26 \Rightarrow P(D_{41}) = \frac{0.26}{1.26} = 0.21; \\
n &= 2 \quad D_s = \text{GDD} \cdot 1.59 \Rightarrow P(D_{42}) = \frac{0.59}{1.59} = 0.37; \\
n &= 3 \quad D_s = \text{GDD} \cdot 2.00 \Rightarrow P(D_{43}) = \frac{1}{2}; \\
n &= 4 \quad D_s = \text{GDD} \cdot 2.51 \Rightarrow P(D_{54}) = \frac{0.51}{2.51} = 0.20.
\end{aligned}$$

The evaluation of Eq. (11.19) yields,

$$P(\overline{S}_4) \approx 10^{-1} \cdot 0.2 \cdot 10^{-2} \cdot 0.5 \cdot 10^{-2} \cdot 0.3 \cdot 10^{-2} \cdot 0.2 \cdot 10^{-1} \cdot 10^{-2} = 0.6 \cdot 10^{-12}$$

This situation is satisfactory and underscores the importance of assessing this type of threat in the design process. The effectiveness of the inspection approach can very quickly make this an unsafe situation, if the damage is not detected.

The examples in this chapter illustrates the importance of making detailed assessment of the different damage threats and situations. The safety of the structure is totally dependent on how the design process solves the residual strength requirements for the total practical design environment and all design situations, and how the inspection programs can be used to compensate for discoveries emerging from the data acquired in service.

11.8. ULTIMATE STRENGTH AND DAMAGE

The evolving practice in composites ultimate strength determination has often embraced the maxim: "If, you cannot see the damage, you have to be good for it." The interpretation comes with some "baggage." First, there has often been a "cut-off" based on an energy value, and then spherical impactors have become a frequently used means for producing "barely visible damage."

The implication is that ultimate strength should be preserved with damage present up to some level that would be detectable, no matter the source of the damage. That raises the first flag, there are not many "spherical impactors" flying around in service nor present on the "ground," in the factory, in the maintenance facilities nor in any other environment flight vehicles are exposed to. A practical definition of realistic impactors would be a first step in achieving a "workable standard." Replacement of "visibility" with "detectability" qualified by some agreed upon probability level would be a constructive addition to the requirements.

A designer's viewpoint of damage tolerance critical structure might take a direction in defining the "ultimate strength damage" that would select the damage size so that ultimate static strength and damage tolerance requirements were equally critical.

Detection depends on whether damage is external, internal or both, and what the method's capabilities are for solely internal damage. There have been many occurrences of sizable internal damage without distinct external indications. So the emergence of economical inspection methods focusing reliably on internal damage would solve many problems, and facilitate the definition of "ultimate strength damage."

11.9. SAFETY AND DAMAGE

Damage size is an important part of safety and therefore very important to the design process. While larger damage means less residual strength, it also means better detection. So it is apparent that a classical case of optimization is at hand. However, residual strength depends on "internal damage size," and detectability depends on both external and internal damage size. Some methods actually favor external damage (e.g. "visual inspection").

The safety of structures depends on detection, as previous situation cavalcade has shown. One message that emerges is that there has to be inspection methods that focus on internal damage, and effectively address a number of situations with growth and very inconsequential signs of damage.

Good, safe design is supported by inspection methods that effectively address detection of (see Figure 11.3),

1. Damage which starts out as minor internal damage size with very faint external signs, and then grows to a very significant threat;
2. Damage which is inflicted in maintenance, remains undiscovered, is hidden from "preflight" inspection, and grows to threatening size;

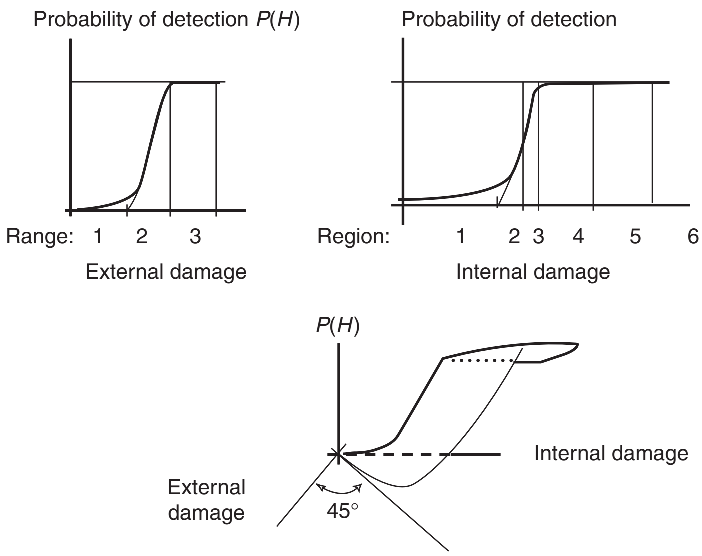

Figure 11.3. Types of detection probabilities.

3. Accidental damage caused by blunt impacting objects that causes almost unnoticeable external damage;
4. Accidental damage inflicted by impactor that causes quite noticeable external damage;
5. Degradation of structural properties without detectable mechanical damage.

In comparing the different inspection methods, it would seem important to favor methods that are sensitive to both internal and external damage sizes. Furthermore, technology development to produce non-destructive methods to discover degradation is a priority, especially for the supersonic environment.

The nature of damage in composites is very different from cracking in metals. Even bonded metallic structures does not match the complexity of composites. The metal approach to corrosion, design by avoidance through protection, also simplifies the damage picture. The requirement of an ultimate residual strength basis for ultimate strength of composites for a presumed hard to detect damage size has brought about a practice that often uses a spherical impactor and external damage as a baseline.

The review of damages in this chapter has made it apparent that a more realistic, practical approach to ultimate strength is needed. The underlying argument has commonly been that "we cannot afford to lose 'ultimate strength' due to a damage that remains undetected, because, it means loss of limit load capability for both fail-safety and damage tolerance to 'active damage fronts.'" This appears to support the inclusion of damage up to "good detectability."

Three alternatives emerge. The first one is to use inspection methods to determine "detectability" that are sensitive to both "external and internal damage," and combine with a criterion of the type, "detectable means to discover the damage 99 per cent of the attempts in an agreed upon test series." The second could involve a criterion of a "reasonable internal damage size" that must be included in the determination of "ultimate strength." The third would be to classify the "total" set of damage threats and to include what internal damage size is compatible with "detectable" for each one.

It is a safety issue that requires us to assure the preservation of fail-safety and other kinds of damage tolerance through the life of the structure. It also is likely that efficient structure will include both metal and composites for a long time to come, so a redefinition of damage tolerance criteria for composites would not eliminate that complication.

In closing, it is important to recognize the importance of detection in establishing desirable levels of safety, and the need for realistic, practical criteria to achieve and preserve them is unconditional.

Chapter 12
Design Philosophy

The nature of ‘‘philosophy’’ evolved in the sixties for designing ‘‘new’’ composite structure was that it had to have, ‘‘equivalent or better level of safety than the structure it replaced.’’ It sounds like a good approach to innovation, and it would be, if pursued without compromise.

The metal world of design, especially the aluminum world, has achieved its safety through ‘‘trial and error.’’ Service experience has had a large influence on existing safety records. The feedback has been preserved in terms of empirical methods, rules of thumb and corporate know-how, and has been very successful.

However, very little from the metal world carries over to composites in terms of ‘‘knowledge base’’ and methods, and most of the ‘‘new’’ safety concerns have a very typical composite quirk. But we must have ‘‘lots of composite experience by now.’’ True for many composite materials (different composite materials) and specific applications. The composites world, though, is in a state of transition. New materials, new processes, new structural concepts are arriving in a steady stream. The technology is in a constant state of innovation.

Historically, the metal quest was a pursuit of ductile, tough, strong, durable, stiff and light materials, and the technology development produced vastly improved properties and superior processes, and served our safety and economy objectives well. But we have reached out further than metal all by itself can take us.

In order to go further, we have been forced to give up some of our measures of ‘‘goodness’’ as at least temporarily unattainable. Ductility was the first to be compromised. The price was ‘‘notch-sensitivity.’’ The typical ‘‘gross’’ allowable for composites often is of the order of magnitude of a third or less of the unidirectional or ‘‘un-notched’’ allowable values. Local effects due to changes in geometry or the presence of undiscovered flaws require attention to minute detail in design.

Composite structure can be designed for many different criticalities. The competing sizing requirements are:

1. Ultimate static strength;
2. Damage tolerance:

    a. Damage with active damage fronts and potential growth; residual strength requirements (limit allowable) prevail;
    b. Fail-safety with loss of a load path; ultimate strength (ultimate allowable value) prevails under limit loads;

3. Damage growth rates;
4. Damage resistance:

   a. Discrete source damage must be contained (for prescribed threats) to a size that makes residual strength adequate for required loads (e.g. for the majority of wing structures $0.7 \cdot$ limit), and in some cases must resist penetration;
   b. Accidental damage must be contained to safe initial size (e.g. contained to region 4).

12.1. ULTIMATE STRENGTH CRITICAL DESIGNS

Federal and International regulations permit the use of *B*-values for fail-safe structure. Criticality of ultimate strength depends on the selection of the damage to be included. A case can be made for equal criticality for ultimate strength and damage tolerance by choosing damage sizes prudently. Design for ultimate strength in the metal world has never required specific damage sizes to be considered in the determination of "Ultimate Allowable Values." For composites, however, current practice and advisory material are emphasizing that, because much damage can be below thresholds of detection, the design must avoid long periods of lost ultimate strength integrity by including undetectable damage in the ultimate allowable values.

So in determining the weight, saving is possible by using composites instead of aluminum undetectable, damage must be considered for composites but not for aluminum because of the differences in detection and the advantages garnered from ductility for the metals. The next example illustrates the competition between ultimate strength critical composite and aluminum structures.

Example 12.1: Allowable values for modern commercial aluminum airplanes have been on a steep improvement slope. So for a skin–stringer wing surface structure, for compression, a typical allowable is ~80 KSI and for tension ~60 KSI.

For composites to compete in weight for a wing surface, with a compression end-load, N_x and an attainable modulus of 12 MSI, the thickness for composites would be,

$$\bar{t}_{\mathrm{C}} = \frac{N_x}{12 \cdot \varepsilon_c}$$

and for metal,

$$\bar{t}_{\mathrm{A}} = \frac{N_x}{80}$$

The comparable weights are,

$$\text{for composite} \quad w_{\mathrm{C}} = \frac{N_x \cdot 0.057}{12 \cdot \varepsilon_c}$$

$$\text{and for metal} \quad w_{\mathrm{A}} = \frac{N_x \cdot 0.10}{80}$$

and for weight ratio, composites to aluminum we have

$$\rho = \frac{0.057 \cdot 80}{12\varepsilon_c \cdot 0.10}$$

which for a ratio of less than 1 (composites lighter), we have

$$\frac{0.057 \cdot 80}{0.10 \cdot 12\varepsilon_c} \leq 1.0 \Rightarrow \varepsilon_c \geq 0.0038$$

and for tension,

$$\varepsilon_{\mathrm{T}} \geq 0.0028$$

So the upper surface of a wing would be marginal in today's technology (0.004), the lower surface and the monocoque of the fuselage would both have an appreciable weight savings, if the structures were ultimate strength critical with a modest damage requirement based on "detectability associated with a damage inflicted by a spherical impactor (a very debatable philosophy and criterion)."

A rational definition of damage to be included in ultimate strength could influence the competitiveness of composites, but it would improve safety levels in ultimate strength critical structure.

12.2. DAMAGE AND RESIDUAL STRENGTH

A successful composite structural design must be based on rational selections of damage sizes. Figure 12.1 describes the "selection variable space," and the nature of a typical situation makes it possible to select a region where the residual strength allowable has a very small slope with regard to damage size (for other situations, the regions need to be smaller).

There are several reasons to focus on the flat region (Figure 12.1). It is a contribution to safety to arrive at a zone where substantial increase in damage size results in only marginal change in residual strength, F_{RS}. It is also common that the probability of detection reaches a plateau for large damage.

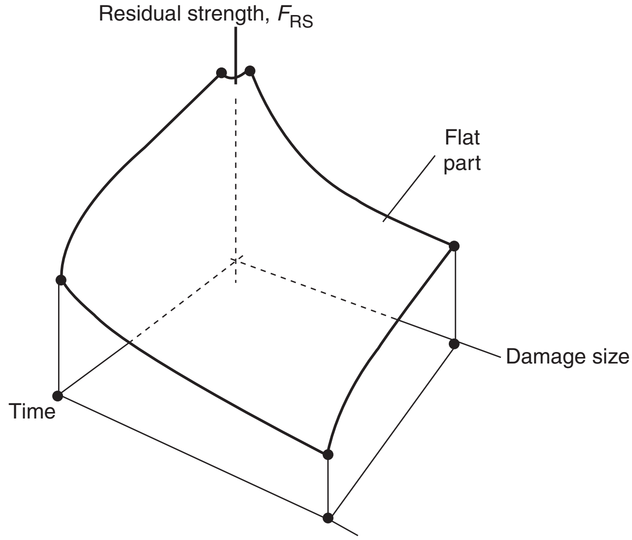

Figure 12.1. Residual strength in stress for a specific detail.

Figure 12.2 illustrates the potential for stable regions of damage sizes both for residual strength and probability of detection, and it would support a good design philosophy to keep this in mind in the material and process selection for the design. Both "flats," (see Figure 12.2), can be approximated as,

$$P(\overline{B}_1 D_5) = \sum_{i=1}^{n} P(\overline{B}_1 D_{5i}) = \sum P(\overline{B}_1 | D_{5i}) P(D_{5i}) \approx p_{CB} \sum P(D_{5i}) = p_{CB} P(D_5)$$

and similarly

$$P(\overline{H} D_5) \approx p_{CH} P(D_5)$$

here p_{CB} and p_{CH} are the constant probability values on "flat," and Figure 12.1 is assumed to represent allowable-like data.

For the case of missing "flats," a larger number of regions must be used in the crucial regimes, and special cases may need special attention for defining the maximum damage criteria, but the philosophy favors exploring the "flat" regions when practical.

12.3. ALLOWABLE AND DESIGN VALUES

Allowable and design values have been the means for preserving structural safety for a long time. Existing regulations permit the use of B-values (90/95), when the

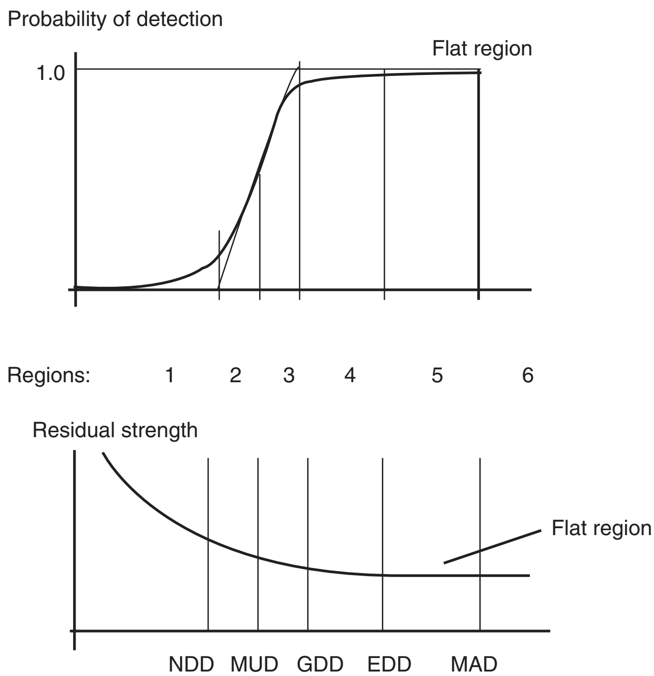

Figure 12.2. Regions of focus for limit loads.

structure is fail-safe (the structure can successfully carry limit loads with one failed load path). Existing regulations also require that "Limit Load" be the largest load expected in service. The use of a safety factor of 1.5 when defining ultimate loads in combination with ultimate *B*-value has assured an excellent structural quality that indirectly has been the cornerstone to safety by supporting structural integrity implicitly.

However, composites, without service experience and sensitive to accidental damage and undetected flaws, need an explicit way to measure and control safety through the structural life. One way to do that is to avoid "Unsafe States" by design, inspection and risk management.

One way is to define the probability of an unsafe state $P(\overline{S})$ and take the steps to keep this probability below some prescribed level.

The probability of an unsafe state can be defined as,

$$P(\overline{S}) = P(\overline{H}_\tau) \cdot P(\overline{X}_T) \cdot \sum_{i=k}^{n} P(\overline{B}_T|\overline{X}_T D_{iT}) P(\overline{H}_T|\overline{X}_T D_{iT}) P(D_{iT}|\overline{X}_T) \qquad (12.1)$$

The following events are included,

$\overline{H}_\tau$: Nothing was found at τ;
$\overline{B}_T$: RS $\leq$ LLR at T;
$\overline{H}_T$: Damage was not found at T;
D_{iT}: Damage size is in region i at T;
$\overline{X}_T$: Damage is present at T.

The index value k in the summation can be chosen to minimize the testing for different damage sizes. The next example, 12.2, gives an illustration.

Example 12.2: The terms in the summation in Eq. (12.1) will be assessed for:

$$n = 5 \rightarrow 10^{-1} \cdot 10^{-3} \cdot 10^{-3};$$
$$n = 4 \rightarrow 10^{-3} \cdot 10^{-3} \cdot 10^{-2};$$
$$n = 3 \rightarrow 10^{-5} \cdot 10^{-1} \cdot 10^{-2},$$

and lower indices are assumed to apply to ultimate requirements. The total value with $k = 3$ then becomes,

$$P(\overline{S}_T) = 10^{-1} \cdot 10^{-1} \cdot 1.2 \cdot 10^{-7} = 1.2 \cdot 10^{-9} \approx 10^{-9}$$

and the only index that would be of importance would be 5. So it seems that, for cases like these, a B-value requirement would be adequate, and if a criteria damage would be used for the sizing, the test requirements for criticality would easily be kept to a practical level.

So a very important part of the design is to strike a balance between the detectability and the residual strength requirements by investigating different detectability situations and find a set of regions that are practically manageable. That would lead to a way to achieve allowables quality data by "joggling" mean values for critical damage and criteria damage.

It is important to get statistical data for residual strength for both ultimate and limit allowables because the presence of damage will tend to make the coefficient of variation larger than what one can expect for undamaged structure.

12.4. ULTIMATE STRENGTH DESIGN VALUES

The "metal world" does not deal with damage for static strength, and the material allowables determination is essentially a statistic evaluation of material properties.

The composites sensitivity to smaller damage sizes has raised the questions, what is the definition of damage that should be included and how should detectability (barely visible damage) be determined (and maybe defined). Previous examples have shown that with a vehicle requirement of "one unsafe flight in hundred thousand" rigorous probability levels must be enforced both on detection and residual strength.

It is also true that any situation that would include undetected damage resulting in loss of ultimate strength also would have caused loss of fail-safe integrity because it is based on ultimate strength of the "remaining structure," and any responsible structural designer would be expected to prevent that from happening.

We will now study a PSE with n load paths and its loss of fail-safe integrity. The focus will be the probability of loss of one load path and the loss of ultimate integrity in another one,

$$P(\overline{S}_{\mathrm{FS}}) = \sum_{k=1}^{n} \sum_{i \neq k}^{n} P\left(\overline{U}_{\mathrm{LP}k}^{\mathrm{L}}\right) P\left(\overline{U}_{\mathrm{LP}i}^{\mathrm{U}}\right) \tag{12.2}$$

The first factor is the probability of loss of limit integrity of load path, k (equivalent to failure, between inspections) and the probability of loss of ultimate integrity of another load path of the PSE. We will use Example 12.3 to explore orders of magnitude for the unsafe state.

Example 12.3: We assume that the PSE is not accessible to walk-around inspections. We also assume that all load paths have equal probability of failure. So Eq. (12.2) becomes,

$$P(\overline{S}_{\mathrm{FS}}) = P\left(\overline{U}_{\mathrm{LP}}^{\mathrm{L}}\right) \cdot (n-1) \cdot \sum_{i=1}^{n} P\left(\overline{U}_{\mathrm{LP}i}^{\mathrm{U}}\right) \tag{12.3}$$

and we assume the terms in the summation are all equal and can be written as,

$$P\left(\overline{U}_{\mathrm{LP}}^{\mathrm{U}}\right) = P(\overline{B}_U | \overline{X} D_l) \cdot P(D_l | \overline{X}) \cdot P(\overline{X})$$

Here D_l represents damage size region l, and the index can take on the values 2 and 3 (1 is taken). A numerical assessment of regions 2 and 3 will now follow,

$$l = 2 \;\Rightarrow P(\overline{S}_{\mathrm{FS}}) = 10^{-5} \cdot (n-1) \cdot 1 \cdot 10^{-2} \cdot 10^{-2} \cdot n$$

which for $n = 10$ becomes 10^{-7};

$$l = 3 \;\Rightarrow P(\overline{S}_{\mathrm{FS}}) = 10^{-5} \cdot (n-1) \cdot 1 \cdot 10^{-3} \cdot 10^{-2} \cdot n$$

which for $n = 10$ becomes 10^{-8}

The question of what size regions should be included under "ultimate strength" could be answered that one through three would result in the following allowable value quality, if three would be the B-value region,

$$P\left(\overline{B}_U|\overline{X}D_1\right) = 10^{-3}$$
$$P\left(\overline{B}_U|\overline{X}D_2\right) = 10^{-2}$$
$$P\left(\overline{B}_U|\overline{X}D_3\right) = 10^{-1}$$

and,

$$P\left(\overline{S}_{\mathrm{FS}}\right) = 3 \cdot 10^{-9} \quad \text{for} \quad n = 10$$

This example illustrates the need-in-detail to determine what damage size regime should be "covered" by ultimate strength requirements, both from an inspection standpoint and an allowable value standpoint.

This example illustrates a process for assessing what size regions to include in the ultimate allowable, but it indicates how a change from "barely visible damage" to "good damage detectability" would support a solid "Fail-safe design philosophy."

The challenge in handling criteria for impact damage in service should be dealt with through realistic assessments of safety and recognition of the real threats present in service, out of which spherical impactors constitute a very modest minority.

12.4.1. Ultimate strength and mechanical fasteners

The challenge with "notch-sensitivity" of composites becomes a very important design issue when mechanical fasteners are used for assemblies, sub-assemblies and details. Open- and filled-hole compression, tension and shear produces substantial reduction in allowable values compared to the un-notched results. This fact puts the presence of holes in the same class of effects as damage. In some contexts, a combination of the two has been used in the design.

In compression cases, one has found that open-hole strength often is lower than filled-hole strength, and it has therefore become one of the more dominant "design drivers," especially when considered in combination with saturation moisture content and maximum temperature.

The loss of ultimate integrity due to "open-hole" effects can be written in terms of probabilities as,

$$P\left(\overline{U}_{\mathrm{U}}\right) = \sum_{i=1}^{m} P\left(\overline{B}_U|\overline{X}_{\mathrm{F}} D_i T_{\mathrm{M}} M_{\mathrm{S}} C_{\mathrm{O}}\right) \cdot P\left(D_i|\overline{X}_{\mathrm{F}}\right) \cdot P\left(\overline{X}_{\mathrm{F}}\right) \cdot P(T_{\mathrm{M}}) \cdot P(M_{\mathrm{S}}) \cdot P(C_{\mathrm{O}}) \tag{12.4}$$

The following events, stochastically independent for damage, temperature and moisture are involved:

C_O: "Open-hole" is critical;
M_S: Moisture saturation;
T_M: Maximum temperature;
$\overline{X}_F$: Damage is present between fasteners;
D_i: Ultimate damage size region;
$\overline{B}_U$: RS $\leq$ ULR;
$\overline{U}_U$: Loss of ultimate strength integrity.

The probabilities involved in this complicated combination of events will be assessed in the next example. A few different situations will be highlighted.

Example 12.4: The first situation involves all effects and considers an ultimate requirement that covers regions 1 through 3.
For $m = 3$, we get:

$$P(\overline{U}_U) = [10^{-3} \cdot 10^{-2} + 10^{-2} \cdot 10^{-3} + 10^{-1} \cdot 10^{-4}] \cdot 10^{-3} \cdot 10^{-3} \cdot 1 \cdot 10^{-1} = 3 \cdot 10^{-12}$$

For $m = 0$, no damage with a late in life moisture content, we get:

$$P(\overline{U}_U) = 10^{-1} \cdot 10^{-3} \cdot 1 \cdot 10^{-1} = 10^{-5}$$

For $m = 3$ and no elevated temperature, we get:

$$P(\overline{U}_U) = 3 \cdot 10^{-5} \cdot 10^{-3} \cdot 1 \cdot 10^{-1} = 3 \cdot 10^{-9}$$

The assessment of these three situations and their orders of magnitude gives an indication that safety considerations in the specific situation would require a detail assessment.

The combination of a fastener hole, maximum temperature, saturation moisture content and damage would constitute a very unlikely situation. Even if prudence would demand an analysis for special cases, it would not be a surprise, if it were ruled out.

The situation with no damage and at a time relatively late in the service life will yield, a probability level that is consistent with good safety levels, a limit extrapolation would yield the order of magnitude of $P(\overline{S}) \leq 10^{-9}$.

A situation with damage and no temperature rise also shows a reasonable safety level.

The conclusion from this type of example is that the realism for different design situations must be assessed for the specifics, so that practical and realistic requirements can be put in place as part of the design process. Safe structural design can only be arrived at after analysis of all the facts.

The whole concept of open- or filled-hole allowables can come into play even for bonded assemblies, if the repair philosophy is such that bolted repairs must be an option, or if damage containment for bonded joints is achieved with fasteners. The next section contains an investigation of situations and probabilities.

12.4.2. Bolted repair philosophy and design requirements

A number of situations are such that the ability to perform bolted repairs on the airplane is considered a definite advantage, and the design consequences are now being considered. PSEs with requirements for bolted repairs must be designed with "open-hole" allowable values. The probabilities of lost ultimate integrity at a design point of the structure is,

$$P\left(\overline{U}_{\mathrm{U}}\right) = P\left(\overline{B}_U M_{\mathrm{S}} T_{\mathrm{M}} C_{\mathrm{O}} R\right) = P\left(\overline{B}_U | M_{\mathrm{S}} T_{\mathrm{M}} C_{\mathrm{O}} R\right) \cdot P(M_{\mathrm{S}}) \cdot P(T_{\mathrm{M}}) \cdot P(C_{\mathrm{O}}|R) \cdot P(R) \tag{12.5}$$

The terminology is the same as used in previous example, and R is the event "the location has a bolted repair." The next example explores the orders of magnitude of the probabilities of participating events.

Example 12.5: It is assumed that n locations are involved at this PSE. It is also assumed that the value is the same for all of them. Eq. (12.5) will be the basis for the evaluation,

$$P\left(\overline{U}_{\mathrm{U}}\right) = P\left(\overline{B}_U | M_{\mathrm{S}} T_{\mathrm{M}} C_{\mathrm{O}} R\right) \cdot 1 \cdot 10^{-3} \cdot 10^{-1} \cdot 10^{-2} \cdot n$$

A comparison with $m = 0$ in Example 12.4 yields, if we want equal criticality for ultimate integrity,

$$10^{-5} = P\left(\overline{B}_U | M_{\mathrm{S}} T_{\mathrm{M}} C_{\mathrm{O}} R\right) \cdot 10^{-6} \cdot n$$

and there are 20 sites for repair at every location, and $P(C_{\mathrm{O}})$ applies to all fasteners at one location. So for this case we could conclude that it would be enough if,

$$P\left(\overline{B}_U | M_{\mathrm{S}} T_{\mathrm{M}} C_{\mathrm{O}} R\right) = 0.5$$

The design approach that deals with provisions for bolted repairs could use "Mean values" for allowable values without compromising safety. If one evaluates the effect on safety (the limit situation), one finds the following value for the probability of an unsafe state.

We assume a normal distribution, $\Phi(t)$ with a $C_v = 0.10$ (based on additional scatter between open-hole and filled-hole),

The allowable becomes $\dfrac{\mu}{1.5}$ and $\Phi\left(\dfrac{(1/1.5)-1}{C_v}\right) = \Phi(-3.33) = 0.5 \cdot 10^{-3}$

The probability of an unsafe state becomes,

$$P(\overline{S}) = 0.5 \cdot 10^{-3} \cdot 10^{-6} \cdot 20 = 10^{-8}$$

So, for the case of "Design for bolted repair" it could be feasible to use mean values.

The importance of a detail analysis of the specifics, on a case-by-case basis, cannot be over-emphasized. Both weight savings and saved safety levels can be the result.

12.4.3. Ultimate strength and allowables

Allowables for composites have gravitated toward strain limits and often have the nature described in Figure 12.3. So if Figure 12.3 illustrates allowable values for different "lay-ups," then it also shows how minimum values for the different direction influences the "acceptable" range. Figure 12.4 shows the nature of the AML (angle ply percentage minus longitudinal percentage) parameter and where, if the strain value is constant along the indicated curve, AML could be an effective

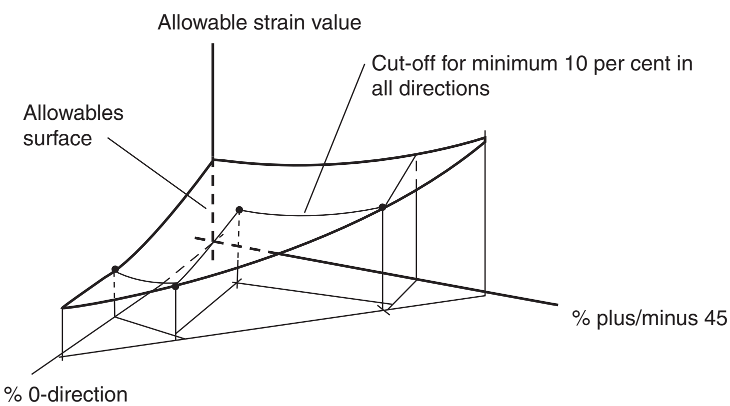

Figure 12.3. Strain allowables surface.

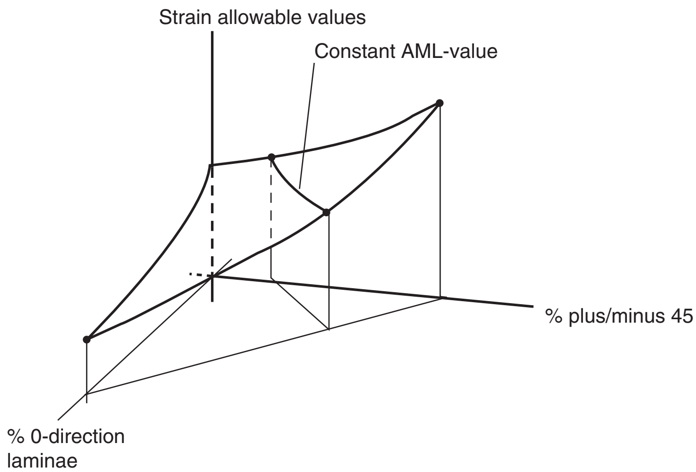

Figure 12.4. Strain allowables and AML (angular minus longitudinal).

concept to use as an independent variable for strain allowables, but only for very specific circumstances.

Present Federal and International standards allow *B*-value (90/95 per cent) allowables for fail-safe (multi-load path) structure. We will now investigate present practices in producing allowables. It is common to use *B*-value stress allowables and combine with some statistics for pertinent "Modulus." Example 12.6 shows order of magnitude for ultimate strength after impact.

Example 12.6: The following probability,

$$\Pr\left(\frac{F}{E} \leq \frac{F_B}{E_S}\right) = p_a$$

is the focus for this example. If we require p_a to be 10 per cent, we would be aiming for *B*-values. It is now assumed that both F and E are normally distributed, and the above equation can be expanded as,

$$p_a = \Pr\left(\frac{F-\mu_F}{E-\mu_E} \cdot \frac{\sigma_E}{\sigma_F} \leq \frac{F_B-\mu_F}{E_S-\mu_E} \cdot \frac{\sigma_E}{\sigma_F}\right) = \Pr\left(\frac{t_F}{t_E} \leq \frac{F_B-\mu_F}{E_S-\mu_E} \cdot \frac{\sigma_E}{\sigma_F}\right)$$

The variables t_F and t_E both have "standard normal distributions" by definition. Hogg and Craig (1972) show that the transformation

$$y = \frac{t_F}{t_E} \quad \text{leads to the Cauchy distribution,}$$

$$F(y) = 0.5 + \frac{1}{\pi} \cdot \tan^{-1} y$$

Our B-value requirement yields,

$$F(y) = 0.10 \Rightarrow y = -0.899$$

and with $C_{vF} = 0.10$ and $C_{vE} = 0.05$, we have,

$$\varepsilon_B = \frac{F_B}{E_S} = \frac{F_B}{1.07\mu_E} = 0.81 \cdot \frac{\mu_F}{\mu_E}$$

The B-value for strain in this example is less than what the intuitive practice would yield. Safety requires that a very rigorous analysis for the specific situation be conducted, in order to comply with the present B-value requirements.

The general purpose of this example is to show the importance of detail case-to-case calculations of B-value allowables in order to maintain acceptable levels of safety.

12.5. DESIGN PHILOSOPHY AND UNCERTAINTY

Innovation is an integral part of design. The perpetual parade of new materials, new processes and new structural concepts in the field of composites makes innovation a very important part of composite structural design. The nature of composites is such that small changes often have substantial impact on the way the design details have to be taken care of. Structural design using composite materials in aerospace is a challenging undertaking, especially from the safety standpoint. Uncertainty is unavoidable, and the management and reduction of uncertainty during service is imperative.

The philosophy of composites structural design and its performance in service can be based on typical situations, if a monitoring system with feedback into a control process, that maintains acceptable safety levels, is in place. This type of risk management can be based on the inspection system.

The difference between producing a database that supports the "typical situation" and producing the base required for "extreme situations" is many times prohibitive in terms of "time and money." The trend in composites structural design has tended to an ever-increasing demand for more test data to deal with increasingly remote possibilities. Existing "Building Block Approaches, BBA," have supported that trend. A future with composites heavily loaded, primary structures in commercial jetliners has produced pressing demands for "safety at a competitive cost."

One answer lies with a philosophy of management of risk and uncertainty, an initial focus on the "Typical Situation" and a monitoring system that produces the feedback necessary to maintain acceptable safety levels.

The design database for the typical situation should be supported by "Scale-up Methods" as complement to the test programs. The technology (see documentation on "Local/Global Analyses," by NASA Langley, Structural Mechanics Department) exists and can be adapted to produce failure predictions for damaged structure based on elements and panels.

12.6. UNSAFE STATE AND DESIGN

The probability of an "Unsafe State," after a major inspection, is the foundation for the design. The inspection approach and the risk value at the end of an inspection period, both interact with the design requirements. If we focus on two inspections τ and T, we can identify three states that are of concern to the probability in question. They are for each PSE:

1. State of Damage;
2. State of Detection;
3. State of Integrity.

If we agree with the philosophy that the structure exposed to accidental damage in service is by definition accessible to "walk-around" preflight inspections, then the focus damage is present at τ and grows into region 5 by T. If, in addition, the aim is to have "sizable" inspection periods, the probability to survive between inspections with lost integrity is essentially nil. The repair policy is assumed to be that, "if detected, repair or 'raise a flag', so it cannot be ignored in the next inspection." With those provisos, the states of interest are:
The state of damage at τ, $S_{\mathrm{D}\tau}$, is:

$$S_{\mathrm{D}\tau} = \overline{X}_\tau D_{4\tau} D_{e\tau}$$

If we stipulate that both detection and integrity depend on the state of damage at the time it is evaluated and that they are stochastically independent. Then we can expand Eq. (12.6) resulting in Eq. (12.7). The probability of an unsafe state can be written for T as,

$$P(\overline{S}_T) = P(S_{\mathrm{D}\tau}\overline{H}_\tau B_\tau S_{\mathrm{D}T}\overline{H}_T\overline{B}_T) \tag{12.6}$$

If we also accept that the regions of damage are selected so that by definition

$$P(\overline{B}_t|D_{6t}) = 1$$

And if the design requirement for growth is:

$$P(D_{5T}|D_{3\tau}) = 0$$

The following equation can be written. The probability of an unsafe state is:

$$P(\overline{S}_T) = P(\overline{H}_T|S_{DT}) \cdot P(\overline{B}_T|S_{DT}) \cdot P(S_{DT}|S_{D\tau}) \cdot P(\overline{H}_\tau|S_{D\tau}) \cdot P(B_\tau|S_{D\tau}) \cdot P(S_{D\tau}) \tag{12.7}$$

Example 12.7 will be used to illustrate an important choice in the philosophy.

Example 12.7: This example uses Eq. (12.7) to assess orders of magnitude. The first evaluation is based on a well-controlled "growth situation"

$$P(\overline{S}_T) = 10^{-3} \cdot P(\overline{B}_T|S_{DT}) \cdot 10^{-2} \cdot 10^{-2} \cdot 1 \cdot 10^{-3} = 10^{-10} \cdot P(\overline{B}_T|S_{DT})$$

As indicated that with this well-controlled damage growth, one could use the "mean" for the allowable value statistics and still arrive at a respectable level of safety (probability of an unsafe state). The outcome would be:

$$P(\overline{S}_T) = 0.5 \cdot 10^{-10}$$

However the situation,

$P(S_{DT}|S_{D\tau}) = 0.5$ instead of 10^{-2} one would get

$P(\overline{S}_T) = 0.5 \cdot P(\overline{B}_T|S_{DT}) \cdot 10^{-8}$ would result in

$P(\overline{S}_T) = 0.5 \cdot 10^{-9}$ if B-values were used for residual strength

The main purpose of this example is to illustrate the design choices between residual strength data quality, damage resistance and damage growth criteria, as all three are important contributors to safety.

It is an important fact that the probability of an "Unsafe State" at T is the best value it will take, and that the probability will continue to grow up to the next inspection. So an important philosophy deals with what the maximum should be allowed to grow to. Clearly, the time between inspections controls how much degradation, damage accumulation and damage growth that will accrue. A balance between different costs (cost of inspection, cost of repair, operating cost of increased weight, etc.) will have to be used in the selection of maximum and minimum values for the probability of an Unsafe State, but with the selection driven by the vehicle safety requirements, and it is hard to believe that an objective of more than one unsafe flight in hundred thousand would be acceptable.

12.7. ULTIMATE INTEGRITY AND DESIGN

Ultimate integrity (a safety factor of 1.5) when the "Largest load expected in service" is defined as limit load, provides an opportunity to develop a fail-safe design philosophy that leads to safety with substantial damage. The traditional fail-safe design approach includes the situation with one load path failed and the remaining structure capable of carrying limit external loads, very often by sustaining internal ultimate loads (due to load redistribution).

The traditional "Aluminum Ultimate, Structural Integrity" is preserved when the remaining structure is undamaged (it retains pristine material strength) and the "redistributed internal loads" do not exceed internal design loads. When it comes to composites, ultimate strength is based on some prescribed damage level. So a comparison of safety levels between metal fail-safety and composites fail-safety would establish a difference in probabilities.

In the "metal world" the "unsafe" fail-safe design would happen with a probability of,

$$P(\overline{S}_{\mathrm{M}}) = \sum_{i=1}^{n} P(\overline{E}_i) \cdot \sum_{i \neq j} P(\overline{B}_j) \tag{12.8}$$

In the "composites world" the probability would be

$$P(\overline{S}_{\mathrm{C}}) = \sum_{i=1}^{n} P(\overline{E}_i) \cdot \sum_{i \neq j} P(\overline{B}_j | D_{Uj}\overline{X}_j) P(D_{Uj}|\overline{X}_j) P(\overline{X}_j) \tag{12.9}$$

The following events are involved:

$\overline{B}$: Strength is less than ultimate requirement;
D_U: Damage is in region 2;
$\overline{E}_i$: Load path i is failed;
$\overline{X}_j$: Damage is present in load path j.

A comparison of orders of magnitude is presented in Example 12.8.

Example 12.8: Eqs. (12.8) and (12.9) are used to demonstrate the numbers involved. The probability of the unsafe metal situation (Eq. (12.8)), with $n = 2$, is

$$P(\overline{S}_{\mathrm{M}}) \approx 2 \cdot 10^{-1} \cdot P(\overline{E}_i)$$

For the composites (Eq. (12.9)) situation, the probability is,

$$P(\overline{S}_C) \approx 2 \cdot 10^{-5} \cdot P(\overline{E}_i)$$

From this comparison it becomes clear that damaged ultimate strength is an overkill for the composites, and pristine ultimate strength would be more appropriate.

These results are an indication that a case-by-case evaluation of all the pertinent facts is absolutely necessary in order to apply a fail-safe philosophy that serves safety. In addition, it is clear that if ‘‘open- or filled-hole strength’’ is critical, the differences in probabilities require detail special assessment of circumstances. One could argue that a good philosophy must include ‘‘hybrid’’ structures (mixture of metal and composite load paths) and an evaluation of relative criticalities, if strength after impact is not critical.

12.8. SURVIVAL PHILOSOPHY

Survival with lost damage tolerance integrity is a very important part of the overall safety situation, but for locations not accessible to ‘‘walk-around’’ preflight inspection, the event must be very unlikely in order to preserve safety. However, for the ones accessible to these inspections, there is a race between detection and failure and safe outcome must occur with a high probability.

It is interesting to ask the question, ‘‘What is the probability, p_k, of not detecting a region 5 damage in k walk-around inspections?’’

k	p_1	p_k
1	0.5	0.5
5		0.03
10		10^{-3}
100		0.00000...
1	0.9	0.9
5		0.6
10		0.35
100		$3 \cdot 10^{-5}$
1	0.1	0.1
5		10^{-5}
10		10^{-10}

This table gives a good indication of what to expect and the importance of worrying about the quality of walk-around inspections.

We are concerned about many situations of damage and failed detection. One important case from the standpoint of design philosophy is the following.

> "A damage size in region 5 is not detected at a major inspection at time T. Integrity is lost. An unsafe state is reached."

The probability of this event is:

$$P(\overline{S}_T) = P(S_{DT}\overline{H}_T\overline{H}_{Tk}\overline{B}_T) = P(\overline{H}_T|S_{DT}) \cdot P(\overline{B}_T|S_{DT}) \cdot P(S_{DT}) \tag{12.10}$$

Our primary interest lies with the event, "An unsafe state is reached and the PSE survives k flights." The probability of the event is:

$$P(\overline{S}_T S_{Uk}) = P(S_{Uk}|\overline{S}_T)P(\overline{S}_T) \tag{12.11}$$

The state of damage is

$$S_{DT} = \overline{X}_T D_{5T} D_{eT}$$

and any potential growth during the orders of magnitude of the k flights, we are interested in, is negligible. Example 12.9 includes a study of orders of magnitude, and Eq. (12.10) is used.

Example 12.9: We start with the state of damage, S_{DT}. We assume that the probabilities of all the external damage ranges are equal, so

$$P(S_{DT}) = 3 \cdot P(\overline{X}_T D_{5T} D_{e3T})$$

We also assume the probability of survival of k consecutive flights can be written as,

$$P(S_{Uk}|\overline{S}_T) = (p_D + \overline{p}_D \cdot p_S)^k$$

If we pursue the design philosophy of aiming for as small as possible probability of not surviving k flights,

$$P(\overline{S}_{Uk}|\overline{S}_T) = 1 - (p_D + \overline{p}_D \cdot p_S)^k$$

Here,

> p_D is the probability of detection;
> p_S is the probability of surviving in an unsafe state.

If we require that $p_D = 0.99$ for the state of damage that is being considered, then we have for the probability of not surviving k flights with an undetected loss of damage tolerance integrity,

$$P(\overline{S}_{Uk}|\overline{S}_T) = 1 - (0.99 + 0.01 \cdot p_S)^k$$

If we assume $p_S = 0.9$ and the probability of survival becomes,

$$p_k = P(S_{Uk}|\overline{S}_T) = 0.999^k$$

The following values result,

k	p_k
5	0.995
10	0.99
100	0.90

It seems that setting requirements for the quality of "walk-around inspections" is an important part of the philosophy of safety and design. However, it is part of the picture to evaluate the realism associated with the, "probability of survival, given an undetected loss of damage tolerance integrity."

The safety associated with a margin under adverse conditions is an important aspect of Design Philosophy that should be evaluated from case-to-case, and the next example contains a parametric evaluation of the challenge.

Example 12.10: The situation is described in Figure 12.5.

The maximum internal loads are assumed to have an exponential probability density function with a controlled probability value, p, between 0.3 and 1.0 (0.3LL

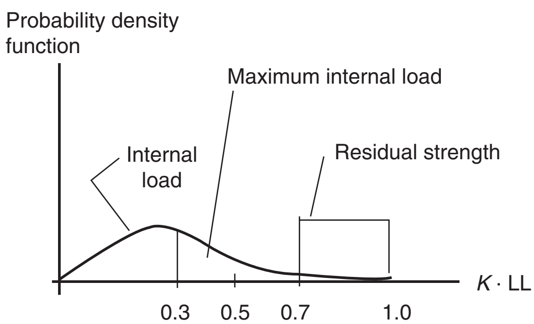

Figure 12.5. Probability density function for internal loads and residual strength after loss of integrity.

and 1.0LL, LL is limit internal loads). The residual strength after lost integrity is uniformly distributed between 0.7 and 1.0LL.
The probability of failure is expressed as,

$$p_f = \int_{r_s=0.7\mathrm{LL}}^{\mathrm{LL}} p_r(r_s) \int_{l=r}^{\mathrm{LL}} p_l(l)dl \cdot dr_s$$

After some manipulations, the result becomes

$$p_f = 0.053p$$

The following results for probability of failure, p_f, and probability of survival, p_s, are listed for different value of p (the percentage of maximum internal load per flight that is between 0.3LL and LL), or $\Pr(0.3\mathrm{LL} < N_{\max} < \mathrm{LL})$,

p	p_f	p_s
100%	0.053	0.947
70%	0.037	0.963
50%	0.027	0.973

The purpose of this example is to illustrate that the probability – of surviving a random flight with an undetected loss of integrity – of about 0.9 is not unreasonable. It could be used a priori to develop some of the quality requirements for the walk-around inspections.

It is interesting to note that even for a relatively low probability of detection of 0.5 during a single inspection, the probability of not detecting an inspectable damage in 10 flights is "small."

It is noteworthy that when "walk-around" inspections cannot be done because of inaccessibility, the safety of the situation is solely controlled by the "low" probability of reaching an unsafe state. The design process would be efficiently served, from a safety standpoint, if the main criterion for achieving "Safety in Service" would be "Keep the 'Probability of an Unsafe State' low."

Chapter 13
Analysis of Design Criteria

The requirements and objectives of a ‘‘new’’ design of composite structure should be analyzed in detail before a rational design process can be started. Traditionally, even including the latest generation jetliners, the safety aspects of structural design has been addressed in terms of factors of safety, margins of safety and allowable–design values with statistical basis and relying heavily on ‘‘Service experience.’’

The next generation commercial jetliners, based on composites innovation, cannot ‘‘lean on,’’ service experience for a large portion of the ‘‘primary structure,’’ including all heavily loaded structures. So, modern innovative designs have to look for other ways to achieve the safety goals required. One way is to establish explicit safety constraints on the structural design process, and that requires numerical, practical, and realistic measures of levels of safety.

These measures should be tied to the overall vehicle safety requirements, a set of special, consistent contributions, so that all the factors that influence vehicle safety can be accounted for. This chapter shows a practical path toward describing how to ensure the structural safety performance in an environment of innovation.

The fundamental measure can be defined by the concept of one unsafe flight in *n* flights founded on the idea ‘‘Undetected loss of integrity.’’

13.1. VEHICLE OBJECTIVE

The overall vehicle safety objective can be formulated in many ways, but it is hard to imagine a case of more direct value than:

‘‘One Unsafe Flight out of 100 000 Flights.’’

We will analyze the detail objectives in the following sections in terms of minimum goals. The 1996 report by the ‘‘Commission on Aviation Safety,’’ chaired by Vice President Al Gore contains a historical account of causes of mishaps and accidents, and concludes that safety incidents traceable to structural problems is of the order of magnitude of 5 to10 per cent for a variety of structural areas.

13.2. OVERALL STRUCTURES OBJECTIVE

Chapter 1 defines the elements of safe structure and Eq. (1.1) expands the safety into four parts. The converse, "unsafe structure," can approximately be expressed as,

$$P(\overline{S}) = P(\overline{D}|IMO) + P(\overline{I}|MO) + P(\overline{M}|O) + P(\overline{O}) \tag{13.1}$$

Eq. (13.1) introduces the second uncertainty into the goal-setting process. The first uncertainty concerns the value of $P(\overline{S})$. The conclusions by the commission, mentioned earlier, are based on the data from fleets with dominantly metal structures. Operations have been updated. The management of airports have been upgraded and is changing. It seems that selecting a 10 per cent initial value and making this uncertainty part of the monitoring and updating would yield a good starting point, especially if it were larger because of the missing service experience it would be a safe approximation.

The first safety objective then becomes,

$$P(\overline{S}) = 10^{-6} \tag{13.2}$$

Eq. (13.1) has four parts. The first term on the right-hand side,

$$P(\overline{D}|IMO)$$

is an expression for "unsafe design," given safe maintenance, manufacturing and operation. It is the focus of this analysis. The target setting depends on the values of all the four terms. The records contain many incidences where maintenance, manufacturing and operation mistakes have played a significant role in accidents. Future feedback will have to be used in keeping a current record of the four effects. The starting point could be set at,

$$P(\overline{D}|IMO) = 0.25 \cdot 10^{-6} \tag{13.3}$$

The value is based on the assumption of an equal share of all the parts. This equation can also be looked at as an expression of the, "probability of undetected loss of integrity of the total structure."

It would constitute the sum of all the probabilities of all the parts; the principal structural elements.

13.3. PRINCIPAL STRUCTURAL ELEMENTS CRITERIA

A principal structural element (an element, the failure of which will result in loss of airplane), PSE, requires its own design criterion. A PSE has a number of potentially critical damage locations, and the relative criticality of the different locations must be determined, especially when "criteria damage" is part of the design.

Criteria damage is in this context defined as damage more critical than the maximum damage by all the potential, realistic threats that the PSE could encounter. The use of this damage for design makes it possible to use mean values and selecting the mean so that *B*-value quality design data results.

Whichever way one chooses to size damage tolerance critical structure, it is necessary to have a rational philosophy for determining, at least, what is the critical type of damage at each location. And if it is possible to vary "thicknesses" from location to location in an independent way, that would answer the question.

One way to determine criticality, for the case with varying thicknesses, could be by using the probability of an unsafe state at the location as the measure of criticality and choosing the damage with largest probability as the most critical damage type. The probability at location i for type j is,

$$P(\overline{S}_{ij}) = P(\overline{B}_{jT}|S_{DijT}) \cdot P(\overline{H}_{Tj}|S_{DijT}) \cdot P(S_{DijT}) \tag{13.4}$$

where

$$S_{DijT} = \overline{X}_{iT}T_jD_{5j} \quad \text{and } T_j \text{ indicates damage type}$$

However, if the objective is to determine the most critical location, one could use the measure,

$$P(\overline{S}_i) = P(\overline{S}_{i1} \cup \overline{S}_{i2} \cup \cdots \cup \overline{S}_{in_{ii}}) = \sum_{j=1}^{n_i} P(\overline{S}_{ij}) \tag{13.5}$$

and choosing the location with the largest probability as the critical one.

One way or another, each PSE would have to satisfy a requirement of a maximum probability of an unsafe state. Assume that all PSEs should satisfy the same requirement, then we would have for PSE k, with a total of, e.g. 50 PSEs,

$$P(\overline{S}_k) = \frac{0.25 \cdot 10^{-6}}{50} = 0.5 \cdot 10^{-8} \tag{13.6}$$

13.4. ULTIMATE REQUIREMENT

Section 13.3 dealt with the criticality for safety critical (damage tolerance critical), structure. Ultimate loads are defined as the loads resulting from applying a safety factor of 1.5 to limit load (in most cases), and limit load is defined as "the largest load expected in service." Translating the conditions of ultimate into probabilities of an unsafe state requires an analysis of the allowable values characteristics.

If we ask the question, "What quality allowable data do we need to avoid 'an unsafe state'?" If we accept the philosophy that if the structure is fail-safe, then we can use *B*-values. A comparison between mean value and *B*-value for normally distributed strength is shown in Example 13.1.

Example 13.1: We will compare requirements for *B*-value and mean. Starting with *B*-values, we have for the standardized normal distribution,

$$\Phi(t) = \Phi\left(\frac{x-\mu}{\sigma}\right) = 0.10 \Rightarrow t = -1.30 \Rightarrow \frac{x}{\mu} = 1 - C_v \cdot 1.3$$

We now ask, "What is the probability that strength is less than $F_B/1.5$?"
Where F_B is the *B*-value and the question could be interpreted to deal with the limit allowable value. The answer is,

C_v	t	$\Phi(t)$
0.05	−7.53	10^{-12}
0.07	−5.62	10^{-7}
0.10	−4.20	10^{-5}

The requirement, if *B*-values are used, is that $C_v < 0.07$.

If we now repeat the same evaluation for the mean value used for allowables, then,

C_v	t	$\Phi(t)$
0.05	−6.66	10^{-11}
0.07	−4.76	10^{-6}
0.10	−3.33	$4 \cdot 10^{-4}$

The requirement, if mean values are used is that $C_v < 0.06$.

If these two requirements are met, we find, it is enough that damage tolerance residual strength value is such that Eq. (13.6) is satisfied. If now a philosophy of preservation of Fail-safe integrity is in effect, then one might wonder, "what it takes" to bring fail-safe integrity in line with the same requirement. There has "always" been an implication in fail-safe design philosophy that detection is expected before next flight after a member fails (a load path is eliminated). So with that in mind, we investigate the following expression for "undetected loss of fail-safe integrity" for a specific PSE at time k,

$$P(\overline{S}_{k+1}) = \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} P(\overline{B}_{Uj}|S_{Djk}\overline{Y}_{ik}) \cdot P(\overline{H}_{j(k+1)}|S_{Djk}\overline{Y}_{ik}) \cdot P(\overline{Y}_{ik}|S_{Djk}) \cdot P(S_{Djk}) \tag{13.7}$$

Here

$$S_{Djk} = \overline{X}_{jk} D_{Uj} D_{ej}$$

The next example, 13.2, will contain an evaluation of orders of magnitude for Eq. (13.7).

Example 13.2: We assume that all the terms are of the same order of magnitude and that $n = 5$ and e is the range two. The following results are practically worth contesting, but the purpose is to illustrate what is needed to support the contention that "mean values are adequate" for fail-safe design,

$$P(\overline{S}_{k+1}) = P(\overline{B}_{Uk}|S_{Dk}\overline{Y}_k) \cdot 10^{-3} \cdot 10^{-2} \cdot 10^{-1} \cdot 10^{-1} \cdot 10^{-2} \cdot n(n-1)$$

which yields,

$$P(\overline{S}_{k+1}) = 2 \cdot P(\overline{B}_{Uk}|S_{Dk}\overline{Y}_k) \cdot 10^{-8}$$

so if mean is used, we have,

$$P(\overline{S}_{k+1}) = 10^{-8}$$

A comparison with Eq. (13.6) could answer the question posed, and it is left to the dedicated reader to make an individual "design judgment."

From an ultimate design criteria standpoint, the whole challenge of selecting the damage size to include in the ultimate design data is very central to achieving a balanced design. An often occurring debate seems to struggle with the difference whether to have a requirement based on barely visible damage, BVID, or on damage with good detectability, GDD.

In either case, it is required to have a thorough knowledge of the in-service inflicted damage types. The relation between external and internal damage sizes and the nature of the threat also must be considered. Of course, the type of inspection method used in the major programs and special considerations for the nature of the walk-around inspection will make a big difference.

From a criteria standpoint, there are two kinds of damage. There are small damage sizes that grow to integrity threats in time, if not discovered. The exterior quite often shows no signs of damage. Consequently, one must use an inspection method that can find internal damage, in order to prevent loss of ultimate integrity. Safety considerations for this case must focus on limiting damage growth rate.

One reasonable criterion could be to require damage in, e.g. region 2 not to grow out of region 3 in three inspection periods. Specific circumstances influencing the objectives should prevail. There also is an accidental damage in service with exterior damage. The expectation is that if the location is exposed to accidents, it is accessible to walk-around inspections. Here, the important design criterion must encourage a damage-resistant design, either in sizing or by reinforcement–protection. The set of criteria therefore must specify "ultimate strength threats" for which the internal initial damage size must be limited to regions 1 and 2, and damage tolerance threats for which initial damage sizes will have to be contained in regions 3 and 4. The relation between external and internal damage must be characterized for all threats and calibrated with the walk-around inspection effectiveness in selection of damage regions and ranges.

The ultimate strength damage threats can be based on a combination of past history and the concerns about evolving phenomena (e.g. undetected hail damage).

The need for damage containment in bond lines (adhesive bond, co-bonded joints and co-cured "bonds," damage), and designs that locally reduce damage growth makes it very difficult to avoid concerns for the ultimate strength of open- and filled-hole details. As bolted repairs also raise the same concerns, it would simplify the detail work if the damage regions were selected so that strength after impact would be more critical. This is especially attractive for damage tolerance critical (safety critical) structure, where it could be done without any weight penalty.

The importance of criticality, the rational selection of damage size regions and ranges, a practical definition of threats and a practical relation between detection and internal damage sizes must be part of the ultimate criterion, especially, keeping in mind the importance of maintaining fail-safe integrity. Fail-Safe Integrity has been the "backbone" of structural safety for a long time and deserves to be adapted to composites innovation.

Finally, it is important for static strength that the design "works around" the weaknesses of composites. The "notch-sensitivity" has already been mentioned. The other important frailty is "low-bearing strength." Both these two sensitivities would favor designs that avoid mechanical fasteners and high load transfer in composites. So a very critical part of the design criteria is the part that advocates the use of adhesive joints and metal structure for carrying concentrated loads and transferring high loads. This part should stipulate requirements that favor efficient designs.

13.5. DAMAGE TOLERANCE REQUIREMENTS

The focus of this section is on limit load requirements with damage. The design criteria must involve a definition of the realistic threats. As limit load is "the largest load expected in service'' it is easy to see that a loss of damage tolerance integrity is a real threat to safety. So the design criteria have to be based on the level of damage resistance that the PSE is designed to possess, and then it has to define the maximum damage growth rate the design will be validated for. The requirements also have to identify the inspection methods to be used and an initial value of the inspection period.

With these preliminaries, it is possible to define region 4 as containing the maximum allowed initial damage size. Exceptions can be made for ''no-growth'' situations, when potentially initial damage in region 5 could be allowed. Region 5 must be defined as the region where the damage size is easily detectable (e.g. probability of detection is 0.999). It should also be defined as a region where residual strength is insensitive to moderate changes to damage size.

The structural concept involved is very important in defining damage types and sizes. At least two are worth mentioning. The first one is, skin–stringers, where the means of assembly can be bonding, co-curing, mechanical fastening or stitching (and whatever the future has in store). Whether the application is wing, fuselage, empennage or special details (e.g. pressure decks, nacelles, etc.), a thorough understanding of the nature of the internal loads is a prerequisite to selecting realistic threats. The second one is, honeycomb surfaces, where the attachment to the substructure plays a large part in identifying a complete set of competing damage types.

If the design will be based on ''criteria damage'' it is possible to develop design data based on means, provided the safety level required does not go further than the *B*-values. The alternative could be to develop *B*-values for the critical damage type and location. As criticality is part of both approaches, the advantages between the two could be different for case-to-case. The basic requirement is satisfying Eq. (13.6). From a simplicity standpoint, ''a legislated damage'' that establishes a basic damage tolerance level is attractive and the optimization that produces a balance between damage tolerance, damage resistance and damage growth will be less constrained. A criterion that considers discrete source damage and the related damage resistance requirements at the same time, if practical, could result in a very efficient design process.

It is important that the criteria for material and structural concept selection consider all three, tolerance, resistance and growth, to establish a favorable starting point for damage-tolerant designs.

13.6. INSPECTION CRITERIA

Many types of damage are potentially part of the safety challenge. The two most difficult types are severe, accidental damage that happen during maintenance or during inspections, not detected and not accessible to "walk-around" inspections and manufacturing growing flaws without external signs of damage. The first type must be dealt with in terms of damage resistance designs that prevent any initial damage larger that region 4, and a quality control that makes the probability of this type of damage very small. The second type requires inspection methods that detect internal damage in region 4 (with external damage in range one) with high probability. The alternative is to design damage containment such that damage does not grow out of region 4. These are special consideration only dealt with on a case-by-case basis either by design, technology development or means that eliminate these types of threats to the structure.

The general cases require that the design criteria specify efficiency of the inspection method. For example, in the cases we have studied, concerning the "minimum safety requirement" of one unsafe flight in hundred thousand, we have found that a reasonable requirement for detection in region 5 is a minimum order of magnitude of the probability of detection of 0.999, provided *B*-value residual strength is used in the design. This is a situation when the design process is best served by range criteria.

We also have discussed the marginal probability of detection and the approach that underlies damage tolerance rating, DTR. We also studied the practice of major manufacturer, and the preferred minimum requirement for "primary structure" of

$$5 < \mathrm{DTR} < 6$$

and how that coincided with what seemed reasonable for composites and a marginal probability of 0.01, which also supported *B*-value usage. In setting the criteria, it is useful to plan the risk management control process function of major inspections, so that there is some manageable range for "the course corrections," the emerging service data may require in order to maintain acceptable safety levels.

The variation of the probability of an unsafe state, as defined in the control process is a good starting point for delineating inspections interval that support the safety level of the design from the start. Figure 13.1 shows the definitions of major inspections and variation of probability of an unsafe state. The figure shows how the probability of an unsafe state varies in time (*as does level of safety*). So if the lower bound, LB is set by design and inspection methods choice, the upper bound, UB can be controlled by the length of the inspection period. The value of the unsafe state at t (between the major inspections at τ and T, and σ precedes τ) can be written as,

$$P(\overline{S}_t) = P(\overline{H}_\sigma \overline{H}_\tau \overline{U}_\tau) + P(\overline{H}_\sigma U_\tau \overline{H}_\tau \overline{U}_t) \tag{13.8}$$

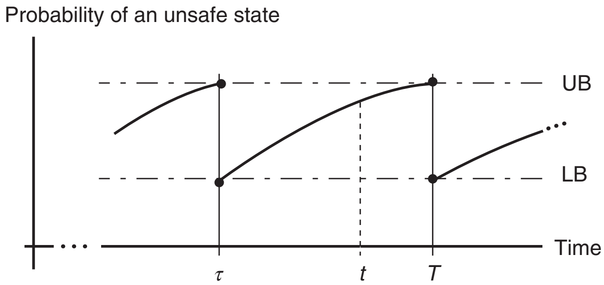

Figure 13.1. Risk management and unsafe state.

The first term describes the probability of no damage detection during the inspections at σ and τ, and lost integrity at τ. The second term describes the same state of detection but with intact integrity at τ but with lost integrity at t.

We can now determine the value for the last flight before the inspection at T and the increase of the probability of an unsafe flight between τ and T can be written as,

$$\Delta_T P\left(\overline{S}_\tau\right) = P\left(\overline{U}_T U_\tau \overline{H}_\sigma \overline{H}_\tau\right) = P\left(\overline{U}_T | U_\tau \overline{H}_\sigma \overline{H}_\tau\right) \cdot P\left(\overline{H}_\tau | U_\sigma \overline{H}_\sigma\right) \cdot P\left(\overline{H}_\sigma\right) \cdot P(U_\tau) \quad (13.9)$$

A study of the orders of magnitude can be found in Example 13.3.

Example 13.3: This example focuses on Eq. (13.9). The first factor on the right-hand side can be considered equivalent to (because of the recommended choice of damage regions and the state of detection),

$$P\left(\overline{U}_T | U_\tau \overline{H}_\sigma \overline{H}_\tau\right) = P\left(\overline{B}_T | D_{5T}\right) P(D_{5T} | D_{4\tau}) P(D_{4\tau}) \quad (13.10)$$

If we set the value of LB to $2 \cdot 10^{-9}$ (compare Eq. (13.6)), then Eq. (13.9)

$$3 \cdot 10^{-9} \geq \Delta_T P\left(\overline{S}_\tau\right) \approx 10^{-1} \cdot 0.2 \cdot 10^{-3} \cdot 10^{-2} \cdot 10^{-2} \cdot 0.5 = 10^{-9}$$

The example then illustrates that using B-values, controlling a "moderate growth," having a prudent definition of regions through the adequate damage resistance and applying a detection–repair criterion that says either:

> "When detected, repair" or "When detected, at least apply a 'rider,' so that the damage is under constant, close scrutiny."

can result in a level of safety that "reasonable men" could consider.

If, when service data emerges, there are updates that require correction in safety level, especially, changing the inspection interval can be an effective way to manage "risk."

13.7. DAMAGE GROWTH RATES CRITERIA

Damage growth often is very dependent on service environments, and substantial uncertainty exists on how rates should be evaluated and controlled. The "community" has minted the expression "no-growth approach," to refer to a no-growth situation. It is, however, rational to postulate a range of growth rates from a minimum to a maximum, a distribution and a feedback mechanism that makes service data active participants. If nothing else, it would avoid the dilemma of "proving a negative" which Philosophy has much to say about, none of it is very helpful to engineering.

The damage size regions become very important in this context, as does the threat definitions. The importance of a thorough and exhaustive threat definition is important, and a "total" array of growth environments must also be part of the design criteria. And if we look at a possible starting point, we can see one in Figure 13.2.

Figure 13.2 shows a case where growth would be controlled for three inspection intervals. It is predicated on a criterion that sets maximum initial, accidental damage size in region 4.

We will now study growth in Example 13.4.

Example 13.4: We assume that damage sizes are uniformly distributed between the curve for maximum growth and the line of zero growth, and the focus is on the

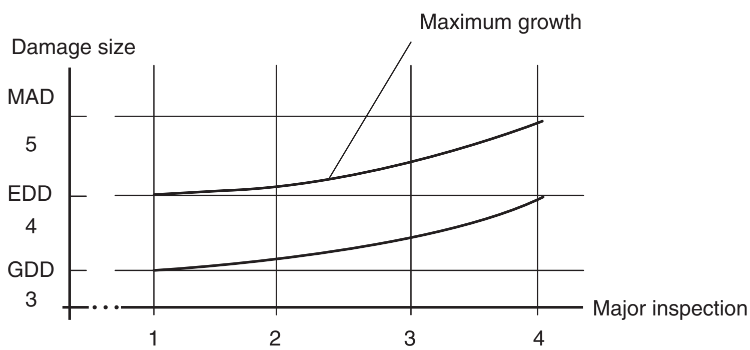

Figure 13.2. Growth between inspections.

probability of not detecting the damage. We assume that the growth from one to two only depends on the size at one, etc. We write the probability of missing a growing damage in four major inspections, as,

$$P(\overline{H}_{14}) = P(\overline{H}_4|S_{D4}\overline{H}_3) \cdot P(S_{D4}|S_{D3}\overline{H}_3) \cdot P(\overline{H}_3|S_{D3}\overline{H}_2) \cdot P(S_{D3}|S_{D2}\overline{H}_2) \cdot P(\overline{H}_2|S_{D2}\overline{H}_1) \cdot P(S_{D2}|S_{D1}\overline{H}_1) \cdot P(\overline{H}_1|S_{D1}) \cdot P(S_{D1}) \tag{13.11}$$

So, the probability of not detecting the growing damage in three periods is,

$$P(\overline{H}_{14}) \approx 10^{-3} \cdot 0.5 \cdot 10^{-3} \cdot 0.3 \cdot 10^{-3} \cdot 0.2 \cdot 10^{-2} \cdot 10^{-4} = 3 \cdot 10^{-17}$$

And in two periods,

$$P(\overline{H}_{13}) \approx 6 \cdot 10^{-14}$$

And for one period,

$$P(\overline{H}_{12}) \approx 2 \cdot 10^{-10}$$

This example illustrates the detectability situation that is associated with control of growth. It seems that a growth rate which changes sizes in both regions 4 and 5 in three inspection intervals, starting from being totally contained in four, seems to be an adequate criterion, if the probability of detection is 0.999 for region 5. It would also produce adequate detectability with zero growth.

Again this example illustrates potential criteria for the combination of damage resistance and damage growth that will have to be determined from case-to-case in order to produce "reasonable criteria" for damage size regions, detectability, damage resistance and growth.

13.8. THREAT AND DAMAGE CRITERIA

There are three states that are important for damage tolerance design. They are:

- State of Integrity;
- State of Damage;
- State of Detection.

They are important in maintaining safety. In the context of design criteria, it is important to identify and characterize threats, which entails dealing with these three states to define what is acceptable and what is not.

There is a continual change in the nature of what constitutes a realistic threat. The procedures at the airports are changing and the traffic intensity is increasing. Traditional "non-events" become threats because of new materials and structural concepts. More activities at the airports (e.g. ongoing construction in many places) have produced new type of threats (e.g. impact by construction debris).

The last few years have produced an array of "new" concerns:

Overlooked damage by exploding landing gear tire fragments;
Undetected peripheral damage by blade fragments from disintegrating engines;
Damage from in-flight impacting hailstones;
Overlooked damage by ground impact of hailstones;
Construction debris "launched" by the exhaust stream;
Ground collision by service vehicles, etc.

These concerns should be considered for addition to the multitude of well-recognized threats that have traditionally been included in the "overall damage tolerance criteria."

Each threat must have its own definition of damage sizes. Figure 13.3 contains nomenclature and influences that are the basis for the definitions of the size regions. Figure 13.3 describes desirable objectives for the damage regions. Regions 1 and 2 would apply to ultimate strength. Region 3 could be left for intermediate requirements between ultimate and limit and 4 and 5 would be defined with limit requirements in mind. Finally region 6 is in the extreme size regime and is not included in the design requirement with the proviso that integrity is compromised and collateral damage could be expected.

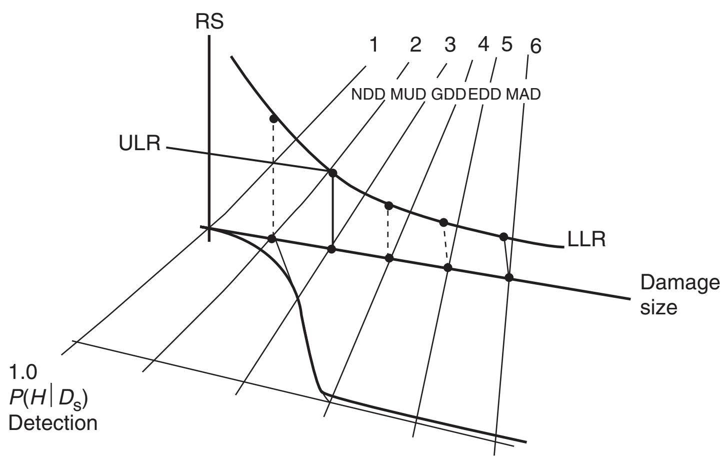

Figure 13.3. Residual strength, detection and damage regions.

For damage tolerance critical structure, one could make the case that the ultimate region should be extended to GDD, as no weight penalty would be involved. Region 4 could be selected so that the damage resistance design would not have initial accidental damage larger than EDD, and consequently, growth would be the only mechanism that produces damage sizes contained in region 5. This would allow a design process that limits growth rates.

One could think of region 5 as consisting of sub-intervals, D_{5i}, and the probability of residual strength being less than required, $\bar{p}_L$ could be expressed as,

$$\bar{p}_L = P\big(\overline{B}(D_{51} \cup \cdots \cup D_{5n}\big) = \sum_{i=1}^{n} P\big(\overline{B}|D_{5i}\big)P(D_{5i}) \tag{13.12}$$

Example 13.5 contains an evaluation of the influence of orders of magnitude of the variation of the terms and factors on the right-hand side of Eq. (13.12).

Example 13.5: The situation, illustrated in Figure 13.3, about constant value of RS, would result in the first factor in the summation term being constant and

$$\bar{p}_L = P\big(\overline{B}|D_5\big)\sum P(D_5) = P\big(\overline{B}|D_5\big)$$

This expression yields a simple definition for allowables work. Now suppose that the variation inside region 5 is noticeable, and suppose that it can be represented by geometric progression so that

$$\bar{p}_L = P\big(\overline{B}|D_{5n}\big)\sum_{i=1}^{n} k^{n-i} \cdot P(D_{5i})$$

Suppose that the distribution of D_{5i} is uniform, we then have

$$\bar{p}_L = \frac{1}{n} P\big(\overline{B}|D_{5n}\big)\frac{1-k^n}{1-k}$$

The value becomes,

n	k	Last factor	First factor times last	Ratio, first–last
5	0.90	4.10	0.82	
5	0.95	4.52	0.90	1.23←
5	0.98	4.80	0.96	1.08
10	0.95	8.02	0.80	1.29
10	0.98	9.15	0.92	1.23←

The noticeable fact (see arrows) is that even for reasonably sizable changes, the *B*-values would only change from a probability of strength being less than 10 per cent to less than 9 per cent. So this type of interval division for the limit range is quite useful. More slope than what has been indicated here should lead to a reassessment of either, the material, the structural concept or the inspection method.

The main purpose of this example is to illustrate the range of usefulness of damage size intervals for damage tolerance design data.

In the selection of threats to be included in the design criteria, it is important to make individual interval assignments for each threat to get a realistic base for the criticality analysis.

13.9. SAFETY CRITERIA BASELINE

Chapter 1 contains a definition of contributions to structural safety seen from a design standpoint. If we now claim that from a criteria standpoint it is important to achieve the totality of,

Safe Operation, O, Safe Maintenance (Inspection and Repair) I,
Safe Manufacturing, M, Safe Design, D and Safe Requirements, R.

The probability of this state can be expressed as,

$$P(S) = P(DIMOR) = P(O|MIRD) \cdot P(I|MRD) \cdot P(M|RD) \cdot P(D|R) \cdot P(R) \quad (13.13)$$

The first factor on the right-hand side is, the probability of Safe Operation, given safe manufacturing, safe maintenance, safe requirements (regulations, criteria, practices, etc.) and safe design.

The second factor is, the probability of Safe Maintenance, given safe manufacturing, safe requirements and safe design.

The third factor is, the probability of Safe Manufacturing, given safe requirements and safe design.

The fourth factor is, the probability of Safe Design, given safe requirements.

The fifth factor is, the probability of Safe Requirements.

The fifth factor could be an important one in an environment of innovation, when there is not a solid experience basis to guide the development of safety regulations. It is an arena where often guidance material substitutes for regulations and negotiations determines regulatory formulations or where enforcement of existing ones is suspended awaiting service data.

The fourth factor is such that it preserves conclusions from Chapter 1. For example the largest load expected in service, limit load, is the foundation of safety and the preservation of ultimate strength is the basis for fail-safety and the practical foundation for preserving ultimate load capability through the life of the structure.

The third factor is the measure of "staying within process and tolerance limits." It should be made part of the monitoring system. Records on MRB actions and in-service discovered, by quality control undiscovered, violations should be collected and the feedback should be used to evaluate the criteria value so that the safety level is preserved.

The same should happen for maintenance and operations mistakes.

The share of an unsafe state "owned" by each of these factors can be derived approximately as,

$$P(\overline{S}) \approx P(\overline{O}|MIRD) + P(\overline{I}|MDR) + P(\overline{M}|DR) + P(\overline{D}|R) + P(\overline{R}) \qquad (13.14)$$

This is another case where, because of innovation, the uncertainty created by insufficient data leads us to use assumptions to initially distribute the individual contributions to the total probability. An initial criterion is required for design, and the monitoring process must be used to update, to continuously test the validity of the initial assumptions.

With no knowledge to suppose something different, it seems natural to assign equal value to the five terms on the right-hand side of Eq. (13.14).

Some examples of the type events that belong to each term are:

$\overline{R}$: Bad "rudder maneuver" regulation;
$\overline{D}$: Failed "fail-safe stabilizer chord";
$\overline{I}$: Bad pressure bulkhead repair;
$\overline{M}$: Wrong post-processing of wing skin;
$\overline{O}$: "Limit load" exceeded.

When it comes to the regulations part of the requirements, there is a lot work to be done. Especially FAR 23 and 25 both need to be updated and uniform in terms of turning advisory material into regulations and achieving consistency. What specifically is needed is safety regulations for composites where the requirements must be levied by objective bodies, while the means of compliance can be negotiated by all the stakeholders based on advisory material.

There are some tenets in structural safety that have had indisputably beneficial effects on the safety records. These should not be abandoned due to arguments of convenience or profit. Only after the display of incontrovertible evidence by the technical community and proof that the last term in Eq. (13.14) is not increased substantially, should changes be considered. Some of these tenets are:

- The cornerstones of damage tolerance;
- The use of an ultimate safety factor of 1.5;
- The use of *B*-values for fail-safe structure;
- Mandatory repair after loss of ultimate strength (based on *B*-values);
- Definition of limit load to be included in the Design Criteria, etc.

This is the time in commercial jetliner history, when safety should be an integral and explicit part of design.

13.10. SCALE-UP CRITERIA

The trend in composites design has been toward larger and larger volumes of design data testing. The building block approach has focused on systems that have multiple levels of testing, picking up the increase in complexity by gradual increase in specimen sizes from coupons to ultimately PSE size test components. In a real world with combined loading, often with pressure, and with many types of damage and environments a "brute test" approach is not practical any more.

A local/global use of the finite element method has shown great promise in replacing many levels in the building block approach, "BBA," with analytical predictions. It would be a great step into the future to establish criteria (regulation supported) that would make it possible to use validated analytical design data. The starting point could be laminate coupons for material allowables for notched configurations establishing statistics for notched and impacted coupons.

As a minimum, there would have to be a level of "panel testing" (and some bolted joint testing), to get a test basis for the added effects of geometry, stress-concentrations, load redistribution and increased scatter due to process variations and tolerances. The scale-up to PSE size structural failures would then be analytical with random test validation. The process would be controlled by criteria (FAA condoned) of checks and balances that assures that the use of the test data produces "validated" analytical failure predictions based on two levels of testing and some selective environmental test support.

13.11. FAILURE CRITERIA

The large majority of aerospace structure is critical in damage tolerance or instability. Although extensive work has been performed (mostly academic) in developing

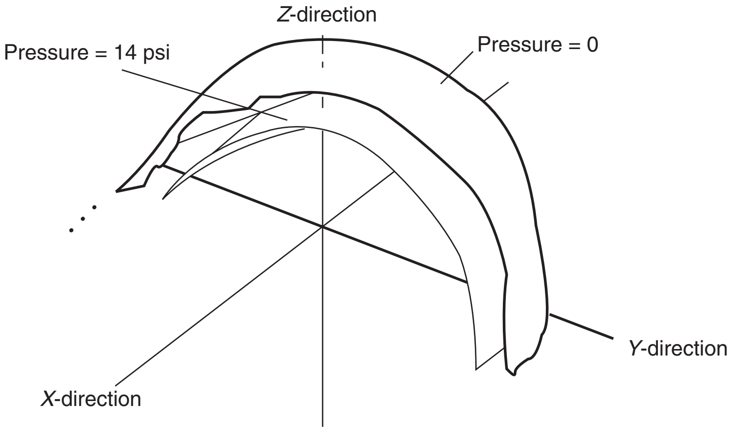

Figure 13.4. Failure surface for three-dimensional internal loads and pressure.

lamina and laminate criteria for pristine structure, there is very little that is useful for the design of practical composite structures. The only success has been produced by the use of empirical criteria. Figure 13.4 illustrates the conceptual needs in the design process. The surfaces in Figure 13.4 represent different pressure levels (like in a pressurized fuselage). Any relation described can be assumed to deal with means of uniaxial internal failure loads and mean pressure. This type of interaction approach has long been used in the design of structures, and could be fit in with the scale-up approach, and could be "derived" from the panel test results, both for instability of pristine and damaged structure for compression, tension and shear.

13.12. MONITORING AND FEEDBACK CRITERIA

There are two objectives of the Monitoring system. It should produce data for risk reduction and for reduction of uncertainty. It also has an educational role by collecting, analyzing and storing data for safe design of future aircraft including derivatives. The educational activities are conducted with the objective of producing a statistical database and a knowledge base for the next airplane, derivative or not, so that risk and uncertainty both are reduced and a safer airplane can be delivered for a reasonable cost.

The first target for the monitoring is to address the question of damage probabilities and support for safety criteria. So the probabilities of interest are:

$$P(\overline{X}), P(D_4|\overline{X}), P(D_5|\overline{X})$$

The following probabilities are included,

Probability of damage;
Probability of damage in region 4;
Probability of damage in region 5.

The service data should be used to update a priori values for each PSE.

The monitoring of data to provide feedback to the aspects of structures is more complicated. Section 13.9 contains an expose of the safety criterion and the contributions to the total. A meaningful feedback for practical updating of the requirements would benefits from two or more levels of detail.

13.12.1. Manufacturing; detail probabilities of criterion

The probability of safe manufacturing can be divided in a set of sub-events for a more descriptive representation. The probability of safe manufacturing, $P(M)$, can be expressed as,

$$P(M) = P(S_M S_P S_I S_A T)$$

The following sub-events are included,

S_M: Within Material specification requirements;
S_P: Within Process specification requirements;
S_I: Within Installation specification requirements;
S_A: Within Assembly specification requirements;
T: Within tolerances.

The detail probabilities can be written as,

$$P(M) = P(T|S_M S_P S_I S_A) \cdot P(S_A|S_I S_P S_M) \cdot P(S_I|S_P S_M) \cdot P(S_P|S_M) \cdot P(S_M) \quad (13.15)$$

Here the first factor on the right-hand side represents, "The probability of safe tolerances, given that all processes have been successful."

The second factor is, "The probability of a safe assembly process, given that all the preceding steps were requirements."

This involves the MRB records and the service records to be useful in re-evaluating this contribution.

The third factor is, "The probability of a safe installation, given successful material processing and that all material properties are within specification requirements."

The same type of data is required for updating.

The fourth factor is, "The probability of safe material processing, given required material properties."

Finally, the last factor is, "The probability of the material being delivered with acceptable material properties."

Variation of this breakdown could be required by special circumstances, but the principle of "detail evaluation" is necessary for useful feedback.

13.12.2. Maintenance; detail probabilities of criterion

Detail maintenance of structure is an important part of safety. A level of sub-events to achieve an effective feedback leads to the following equation

$$P(I) = P(I_R|I_M I_I) \cdot P(I_M|I_I) \cdot P(I_I) \tag{13.16}$$

The following sub-events are included.

The first factor of the right-hand side is, "The probability of a safe repair being performed, given safe maintenance and safe inspections."

The second factor is, "The probability of safely performed maintenance, given that inspections are safe."

Finally, the third is, "The probability of safe inspections."

A breakdown along these lines would make a simple feedback system feasible.

13.12.3. Requirements; detail probabilities of criterion

The probability of safe requirements is especially pertinent during innovation, especially when never-before used materials, processes and structural concepts are introduced. The risk of both missing regulations and advisory material is never larger. It therefore is necessary to include these considerations into the safety assessments. The total probability of safe requirements can be broken down as,

$$P(R) = P(C_A|C_R C_P) \cdot P(C_R|C_P) \cdot P(C_P) \tag{13.17}$$

The sub-events are,

C_A: Safe criteria;
C_R: Safe regulations;
C_P: Safe practice and advisory materials.

This is a very important aspect of safety and difficult challenge because of often swiftly moving technology development. The feedback for safety updates is very important because of the potentially far-reaching consequences of systemic problems.

13.12.4. Operation; detail probabilities of criterion

The effects of operational mistakes can be very damaging to structural integrity, and the feedback aimed at keeping the requirements current must be supported by all government agencies because of the long and arduous feedback channels. The breakdown of the total probability of safe operation can be done like this,

$$P(O) = P(F_M|E_T E_P H_P) \cdot P(E_P|E_T H_P) \cdot P(E_T|H_P) \cdot P(H_P) \tag{13.18}$$

The sub-events are,

F_M: Safe flight procedures employed;
E_T: Safe training of flight crew;
E_P: Safe emergency procedures followed;
H_P: Good health of the flight crew.

The first factor on the right-hand side of Eq. (13.18) is "The probability of safe flight procedures are used, given a well-trained crew, safe emergency safety procedures are used and a healthy crew is in place."
The second factor is, "The probability that safe emergency procedures are used, given a well-trained and healthy flight crew."
The third factor is, "The probability of a well-trained crew, given that they are healthy."
The fourth factor is, "The probability of having healthy flight crews on-board."

The breakdown of the operation into detail factors makes it possible to structure the feedback and analyze the data so "current" evaluations of safety can be made continuously.

The whole idea about producing feedback, about other aspects of flight vehicles than design, deals with the need of having a current assessment of what the safety objective truly is for the structural integrity through the life of the structure. In the process, it becomes clear that safety is the result of interaction between many aspects of service.

13.13. OPEN-HOLE COMPRESSION CRITERIA

An often competing aspect of design of compression critical structure is "open-hole compression." The concept of an open-hole situation comes from two sources, the use of mechanical fasteners for sub-assemblies and assemblies and the fact that non-interference fasteners often mimic the behavior of open holes, under load, and the second source is the need to allow "bolted repairs."

Both cases involve a combination of events that often would be considered very unlikely. The sub-events that are considered include the following probabilities,

B_U: RS > ULR;
H_O: Open-hole response is in effect;
T_M: Maximum temperature has been reached;
M_S: Moisture equilibrium has been attained;
M_V: Basic material scatter prevails.

The probability that ultimate strength integrity has been violated is,

$$P(\overline{U}_U) = P(\overline{B}_U H_O T_M M_S | M_V)$$

Which also describes the probability that the strength, for this case, is less than the prescribed allowable, and the probability can be expanded as,

$$\begin{aligned} P(\overline{U}_U) &= P(\overline{B}_U | H_O T_M M_S M_V) \cdot P(M_S | T_M H_O M_V) \\ &\quad \cdot P(T_M | H_O M_V) \cdot P(H_O | M_V) \end{aligned} \tag{13.19}$$

The first factor, p_A is, "The probability that strength is less than the ultimate requirement, given an open-hole response, maximum temperature, equilibrium moisture content and basic material scatter at room temperature."
The second factor is, "The probability that equilibrium moisture content has been reached, given maximum temperature and open-hole performance."
The third factor is, "The probability that maximum temperature has been reached, given 'open-hole response'."
Finally, the fourth factor is, "The probability of 'open-hole response'."

It is a not so uncommon practice to assume that room temperature scatter applies to the elevated temperature situation. Eq. (13.19) will be analogous (M_V is eliminated), if the assumption is dropped, but the allowable will most likely be less, as scatter tends to increase with temperature. Example 13.6 contains a study of orders of magnitude for what could be considered a "practical" situation.

Example 13.6: Eq. (13.19) is the focus of this example. The first part addresses the situation when mechanical fasteners are only used for "bolted repairs." The probability of less than ultimate strength is,

$$P(\overline{U}_U) = p_A \cdot 0.9 \cdot 10^{-2} \cdot 10^{-2} = 0.9 \cdot p_A \cdot 10^{-4}$$

which if mean were used for allowable value, we would have

$$P(\overline{U}_U) = 4.5 \cdot 10^{-5}$$

This result would represent a much smaller probability than normally is used for ultimate design, and the use of mean values should be seriously considered for this kind of situation. When mechanical fasteners are used prolifically in the structure, the following results,

$$P(\overline{U}_U) = p_A \cdot 1 \cdot 10^{-2} \cdot 0.2 = 2 \cdot p_A \cdot 10^{-3}$$

which for mean value allowable would become,

$$P(\overline{U}_U) = 10^{-3}$$

This case needs a lot more analysis. If we assume a normally distributed strength, the use of a mean allowable value would become very dubious for coefficients of variation in the range of 0.15 or more.

It is interesting to note that mean allowable values from normal strength distributions result in the following "limit allowable probabilities."

It is assumed that,

$$F_{ult} = \mu$$

This assumption results in,

$$F_{lim} = \frac{\mu}{1.5}$$

The normal distribution yields,

$$t_L = \frac{(\mu/1.5) - \mu}{\sigma} = -\frac{0.333}{C_v}$$

which for a few values of C_v produces,

C_v	t_L	$\Phi(t_L)$ = probability of strength below "Allowable"
0.05	−6.66	$< 10^{-10}$
0.10	−3.33	$4.3 \cdot 10^{-4}$
0.15	−2.22	0.013

This table illustrates the range of "limit loss of integrity" when critical for ultimate static strength, and for a coefficient of variation of 0.06 or less, the achieved safety level is in the range of what could be considered acceptable.

If we now investigate the joint event,

$$\overline{B}_U \overline{H}_O T_R M_S$$

where

$\overline{B}_U$: S $\leq$ ULR, strength is less or equal than ultimate requirement;
$\overline{H}_O$: Open hole response is not exhibited;
T_R: Room temperature or less;

which can be interpreted as the filled or partially filled, wet, room temperature case, U_{URT}. We assume for this case that the strength is on an average 25 per cent higher than F_A, and that the varying states of response result in $C_v = 0.08$.

We now assume that the "hot-wet allowable" was set as the mean, $F_A = \mu$. We also assume that we are dealing with a ubiquitous fastener situation. With the normal distribution, the following can be stated for the probability of loss of limit integrity,

$$\Phi\left(\frac{(\mu/1.5) - 1.25\mu}{\sigma_{FH}}\right) = \Phi\left(-\frac{-0.47}{0.08}\right) \approx 10^{-7}$$

which clearly is short of the requirement, 10^{-9}. The ultimate allowable value for the "hot-wet" case must be set at $F_A < 0.94\mu$, in order to reach the desirable level of safety (probability of an unsafe state).
This value corresponds to the following probability,

$$\Phi\left(\frac{0.94\mu - \mu}{\sigma}\right) = \Phi\left(\frac{-0.06}{C_v}\right) = 0.16$$

which hardly appears to make enough difference to justify arguing for *B*-value relief.

This is an example, and it illustrates the need for considering the "total picture." When arguing safety issues only based on the extreme circumstances, a situation that in total is unsafe could "slip by."

This example makes the point that if we use unlikely situations to define allowable values. We need to make detail analysis of the situation, a requirement in the design criterion. The design criteria should contain numerical requirements addressing how to deal with this case. It is especially important as modern aluminums for compression applications often successfully compete with composites with reduced allowable values, like "hot-wet-open-hole *B*-values."

Chapter 13 discusses the importance of design criteria that have a firm foundation of practicalities and are based on a technical analysis with engineering justifications and scientific foundation. "Seat-of-the-pants" engineering, however admirable, has a place in composites and innovation only when "anchored" in rational judgment.

13.14. CRITERIA FOR SAFE DESIGN OF DAMAGED STRUCTURE

A safe design of composite structure can only be achieved if the Design Criteria contains "realistic damage threats" that are location dependent and come with practical "detectability" definitions both in terms of damage size versus probability of detection and in terms of quality requirements (e.g. damage tolerance rating). Accidental damage in service is limited to areas accessible to "walk-around" inspections and must therefore be subject to damage resistance design that limits the initial damage size to region 4 ($d_s <$ EDD). If that limit is not achieved, a safe state would not be attainable with any reasonable probability.

It is possible to have undetected severe damage inflicted during maintenance in locations that are not accessible to walk-around inspection. Therefore, one has to control maximum damage growth by detail design; either damage containment (stitching across bond-lines), protection from severe environmental effects or "heavier" structure (stress reduction). The maximum damage growth rate, e.g. could be required to be such that the probability of growth from region 4 to region 5, in three inspection periods, would be equal to 0.5.

The probability of an unsafe state after the first inspection after impact,

$$P(Y_1 D_{41} D_{52} \overline{H}_1 \overline{B}_2 \overline{H}_2) = P(\overline{B}_2 | D_5) \cdot P(\overline{H}_2 | D_5) \cdot P(\overline{H}_1 | D_4) \cdot P(D_5 | D_{41} Y_1) \cdot P(D_{41} | Y_1) \cdot P(Y_1) \tag{13.20}$$

The next Example 13.7 contains an assessment of "reasonable" orders of magnitude.

Example 13.7: This example focuses on Eq. (13.20), and the probability of an unsafe state is for the following assessment,

$$P(\overline{S}_2) = 10^{-1} \cdot 10^{-3} \cdot 10^{-2} \cdot 10^{-1} \cdot 10^{-2} \cdot 10^{-1} = 10^{-10}$$

By analogy in Eq. (13.20), we can write for the second inspection,

$$P(\overline{S}_3) = 10^{-1} \cdot 10^{-3} \cdot 10^{-3} \cdot 0.3 \cdot 10^{-2} \cdot 10^{-1} = 0.3 \cdot 10^{-10}$$

And if we assume that growth drivers are different in different inspection periods, we have that,

$$P(\overline{S}_3) \approx 10^{-9}$$

So, against the background of stable exponential growth for three inspection periods, we find that controlling the maximum growth, as described, preserves an acceptable level of safety. We also know that after the third period we have,

$$P(\overline{S}_4) \leq P(\overline{S}_3)$$

because of the fact that

$$P(\overline{S}_4) \leq P(D_{41} Y_1 \overline{H}_1 \overline{H}_2 \overline{H}_3 \overline{H}_4) \leq 10^{-12}$$

A growth design that controls maximum growth rates to the levels shown here would then adequately maintain level of safety to acceptable levels with this type of damage.

This illustration attempts to justify the need to make growth rates an important aspect of dealing with accidental damage in location where walk-around inspections are not practical.

Efficient, explicit safety-based design criteria is very important in the design of composite structures. Both the lack of service experience and a very slow evolution of regulations for composites require a meticulous development in the face of innovation; which includes regulations that appear to have a general flavor, but have never tested in the new composite world.

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Chapter 14
Design Example

Design of composite structure involves the choice of materials and structural concepts. The choice of material is often determined by strength and toughness. The process and structural concepts selections are dominated by an array of financial (production and life cycle costs) and performance considerations. Inspection costs and effectiveness are an integral part of both.

The selection of structural concepts still seems to be focused on choices between skin–stringer panels and sandwich panels. The variety of attachment methods (to the sub-structure) and structural enhancements approaches (e.g. stitching) are in dynamic development. So, the approach to design must incorporate explicit, safety-based features to deal with the uncertainty and risk associated with innovation.

The design of composite structural concepts quite often involves an investigation of whether "damage tolerance" or "static strength" is critical. Criticality in design, in many cases, is based on a situation, when the safety factor of 1.5 can be used to separate limit and ultimate internal loads. This is based on an assumption of a linear relation between external and internal loads. There are exceptions, and some are noteworthy.

14.1. GEOMETRICALLY NON-LINEAR STRUCTURAL DESIGN

The typical situation for geometrical non-linearities is described in Figure 14.1. What is striking here is that while the limit external load is described by,

$$P_{\text{Lim}} = \frac{P_{\text{Ult}}}{1.5}$$

the internal limit load is described by,

$$N_{\text{Lim}} < \frac{N_{\text{Ult}}}{1.5}$$

If we write the criticality comparison as a comparison of the effects of maximum internal limit loads, then we can use Figure 14.2 as an illustration of damage tolerance critical structure.

B/k represents the situation in Figure 14.2. The probabilities can be written as,

$$\Pr(N_{\text{Lim}} \leq F_{\text{RS}} \cdot \bar{t}\,) \leq \Pr(N_{\text{Lim}} \leq \frac{F_A}{k} \cdot \bar{t}\,) = \Pr(N_{\text{Ult}} \leq F_A \cdot \bar{t}\,)$$

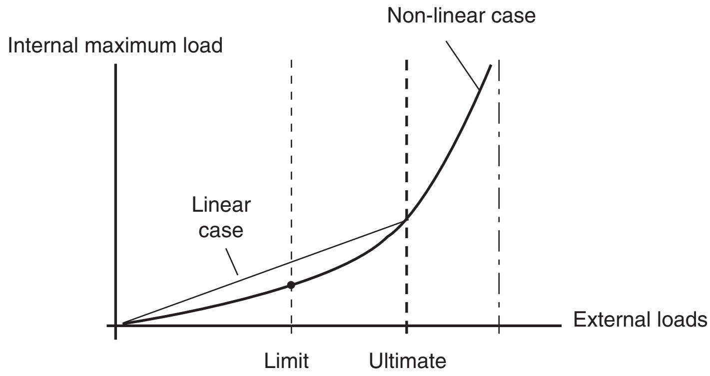

Figure 14.1. Non-linear structural response.

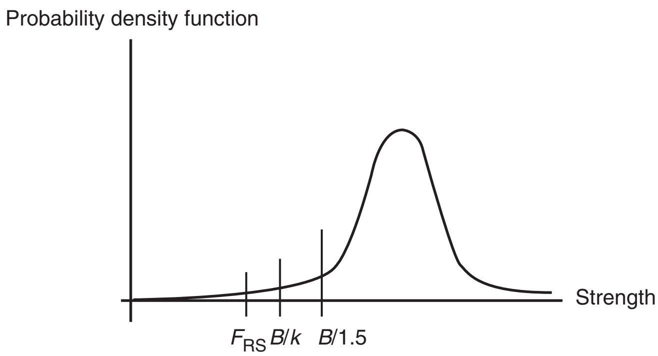

Figure 14.2. Residual strength and corrected *B*-value (away from ultimate).

which is another way of saying that damage tolerance is more critical than ultimate strength, and an independent sizing would result in,

$$\bar{t}_{\mathrm{DT}} = \frac{N_{\mathrm{Lim}}}{F_{\mathrm{RS}}} \geq \frac{N_{\mathrm{Ult}}}{F_A} = \bar{t}_{\mathrm{Ult}}$$

where "residual strength" with structural damage would be the "driving" mechanism. It is clear that any analysis of criticality involves a thorough knowledge of the nature of the internal loads, and non-linear situations deserve special attention. It can be debated whether damage tolerant critical structure is a proper objective for this case, but practicality of the inspection methods and inspection frequencies may force the issue. The practically acceptable "detectability" may not be optimum from a safety standpoint, or special damage threats may require damage tolerance driven designs.

The states of internal loads for aerospace structures are often described as "tension structure," "compression structure" and "shear structure." The latter is often the one that has a dominant type of loading. The other two often present very complex situations, in which the allowable values determine whether they are compression or tension critical. The internal loads of fin structure, by definition, have total reversal. The horizontal stabilizer, often has a close to total reversal of the "balancing tail-load" which tends to make the same thing true for the aft fuselage. Gust critical structure also have reversals. The negative maneuver case for the wing is known to have caused as much as a 60 per cent reversal. Just to mention a few examples. Polymeric composite tends be more sensitive to compression than tension loads (exceptions exist) but a thorough design, with explicit safety constraints, requires meticulous consideration of all three, compression, tension and shear, at most "design points."

14.2. FAIL-SAFETY, MATERIAL NON-LINEARITIES AND HYBRID DESIGN

Fail-safe design has been an important part of safety of metal structure, and can have a vital role for composite structure. A troublesome case is the hybrid structure which, in some cases, comes about in design of structures that need to benefit from both metals and composites to be efficient and effective. The complication originates in the potential for material non-linearities in the metal parts. A traditional case to illustrate the situation can be found in Example 14.1.

Example 14.1: This example deals with the case of three equal load paths, in which case the structure could lose one load path and still carry limit load, $\frac{2}{3}$ of ultimate load; a seemingly desirable situation for an axial case, but, if one of the members is a metallic load path, caution must rule. Figure 14.3a illustrates the details. The ultimate internal loads situation can very well occur under external limit load, which means that the metal part could be far into the plastic range.

Even the "undamaged" structure requires a caution in the establishing of the limits for allowable internal loads. The ratio between ultimate internal loads and limit internal loads can be quite different from the required 1.5 factor for ultimate external loads. The purpose of this example is to reinforce the need for caution in the design of "hybrid structure."

The intact case shows that the ultimate strains in the composite members exceed 1.5 times limit, illustrating the problem with hybrid structures and the caution that is required in design and criteria.

Figure 14.3b describes a three load path structure with a composites load path failed. The illustration emphasizes that the "practice to use linear internal loads" to

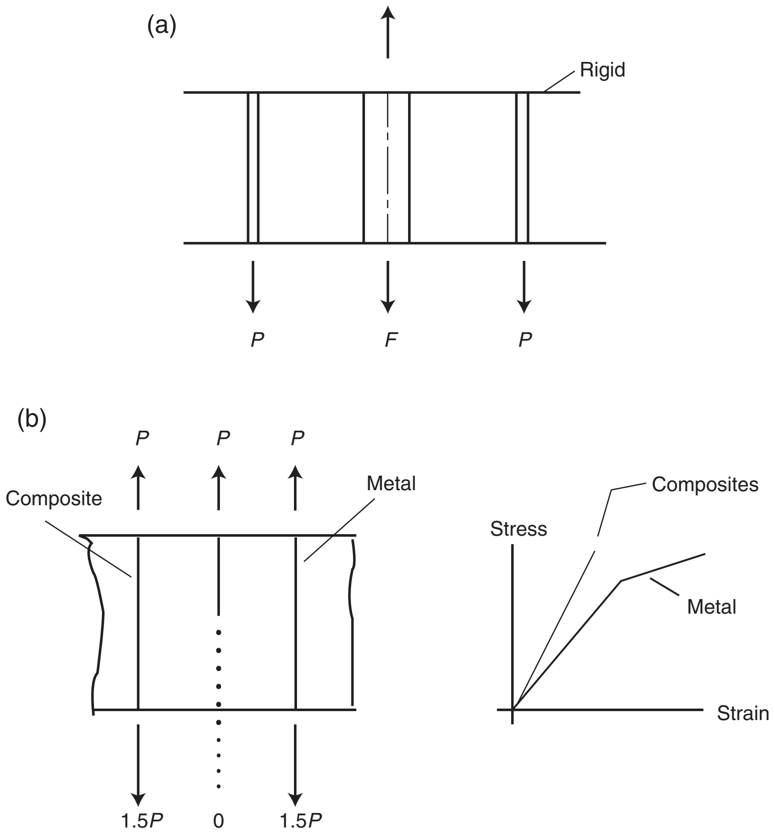

Figure 14.3. (a) Hybrid structure with central metallic member (b) Fail-safe load paths and properties.

satisfy limit requirements continues to be wrong for hybrid structure. The purpose of this illustration is to reinforce the need to have design criteria that requires a detailed definition of what "limit load" means for composite structures with complications. The modulus of elasticity is for:

The composite path: 12 MSI; and
The metal path: 10 MSI;

and the "plasticity modulus" is 4 MSI.

We assume that the original was based on not exceeding the proportional limit and that also is the allowable strain value. After failure, the metal load path carries,

$$P_M = A \cdot \varepsilon \cdot 10^4 + A \cdot (\varepsilon_{CR} - \varepsilon) \cdot 4 \cdot 10^4$$

And the composite load path carries

$$P_{\mathrm{C}} = A \cdot \varepsilon_{\mathrm{CR}} \cdot 12 \cdot 10^3$$

The total load is,

$$P_{\mathrm{Tot}} = A \cdot \varepsilon \cdot 10^3(10 + 24)$$

So the fail-safe case with limit external loads yields the following maximum strain,

$$\frac{\varepsilon_{\mathrm{CR}}}{\varepsilon} = 1.75$$

which obviously is not acceptable, especially as the load introduction for composite structures will involve metal combinations, at least until the composite technology has caught up with all the detail aspects of structural design. In the meantime, non-linear situations must be handled with caution, especially, in the definition of ultimate internal loads.

14.3. FAIL-SAFE CRITERIA IN DESIGN

The ability to select different moduli of elasticity for different parts influences the adaptation of metal fail-safe criteria to composites. Example 14.2 illustrates one aspect of load redistribution. The use of stiffer stiffeners than skins causes a lot more axial load to be carried by the stiffeners than by the skin. The failure of a stiffener in composite structure is therefore, a much more dramatic event than in the metal world.

Example 14.2: This example deals with the relatively challenging design problem associated with the loss of a stiffener with large modulus of elasticity. Figure 14.4 describes the details. The challenge in this type of fail-safe design is that the high modulus of elasticity we aim for in the stringers makes the size of the load to be redistributed large. It is possible that the skin will have a larger strain to failure because of the lay-up, but as the numbers show, it is a difficult balance game, even with average values. We will now investigate the "failed case."

The central stringer is failed. There is as much material in skins as in stringers. The skin modulus of elasticity is assumed to be 4 MSI and the stringer modulus is assumed to be 16 MSI.

The load in the stringer is:

$$P_{\mathrm{str}} = 0.50 \cdot s \cdot \bar{t} \cdot 16 \cdot 10^3 \cdot 0.004 = 32s\bar{t}$$

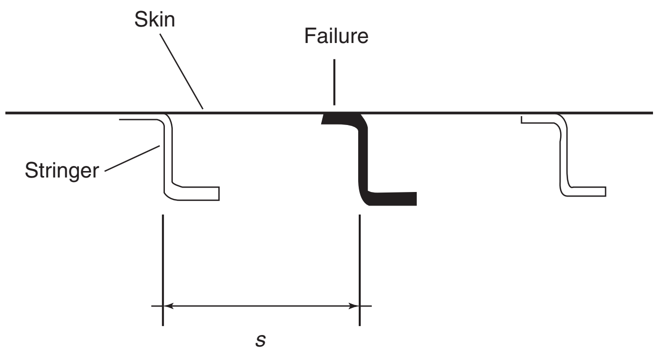

Figure 14.4. Skin–stringers, one stringer failed.

The average strain in the skin in the bay of the failed stringer is estimated to be,

$$\varepsilon_{\text{ave}} = \frac{0.004}{1.5} + \frac{32s\bar{t}}{2st \cdot 4 \cdot 10^3} \approx 2.7 \cdot 0.004$$

while, in a similar case for aluminum, the critical strain would only be,

$$\varepsilon_{\text{ave}} \approx 1.17 \cdot 0.004$$

This is a difference worth taking note of, as accidental damage is one of the most demanding design criteria for composites.

A detail design of structural concepts like the skin–stringer analysis has shown the importance of damage tolerance. Example 14.1 shows the challenge of hybrid structure. A strategy for wing upper surface with composites, except for the vent-stringers, e.g. demands strict attention to detail or fail-safety could become a very difficult problem. Example 14.2 focuses on the classical example of one load path failed and the ability to redistribute the loads. Both cases, point to the need of doing extensive investigation and "target-setting" that includes,

Damage resistance;
Damage growth rates;
Static strength;
Fail-safety;
Damage tolerance in general.

In order to achieve a balanced design of the structural concept used for the PSE in question. It seems right to penalize mechanically fastened concepts for "open- or filled- hole" reductions. It does not seem right to penalize stitched concepts for "open-hole" reductions. It seems that a "stitched repair" should be part of the design objective, or "obsolete criteria" will hamper progress. So in order to enter the damage tolerance design space with a feasible candidate, the concerns about non-linear responses and ultimate strength must have been taken care of and workable repairs must be part of the design criterion.

14.4. STRUCTURAL CONCEPTS AND DESIGN SPACE

A number of key issues in the overall structural design (like major load paths, etc.) are resolved as part of the configuration work. The initial work makes it possible to assess the challenges associated with the PSEs.

The starting point for any design of a PSE is the detail selection of structural concepts. The composite world seems to have focused on skin–stringer panel and sandwich panel concepts, both with a variety of detail configurations and attachment approaches (to the sub-structure).

Design of composite structure must start with a rational set of decisions leading to the selection of a structural concept including commitment to typical details. Unlike the aluminum world, where riveted skin–stringer constructions have dominated for a long time, the design process does not embark on a "well-trodden path," but enters a trail where the design process is "custom-made" to fit innovation, new challenges and explicit safety constraints.

14.4.1. Skin–stringer design space

Figure 14.5 sets the stage for the investigation of skin–stringers. Here the stringer cross section, area, height, spacing and modulus together with thickness and modulus of the skin and attachments are selected to satisfy the ultimate requirements. A failure surface on the panel level (presently of empirical origin) will make the first cycle, e.g. possible.

The selection of type of cross section depends on the "attachment between skin and stiffeners." When the attachment is provided by a bond-line, it has often been found that symmetrical section performs better in stability than the non-symmetrical because of "secondary" deformations. If we are dealing with a panel that carries compression and shear loads, the initial candidate will emerge from a combination of buckling and fail-safe considerations. The dominants, tension case, unlike the metal challenges, often presents an easier path for finding the initial candidate for final design.

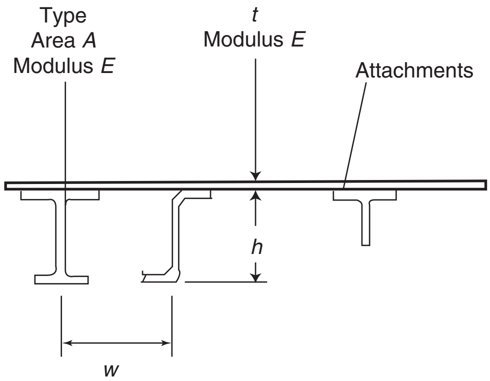

Figure 14.5. Skin–stringers, design variables.

14.4.2. Honeycomb panel design space

The honeycomb concept often presents many "structural" advantages, but also often turns out to be a difficult inspection challenge. Figure 14.6 shows the detail considerations for the design of the concept.

The loads, including pressure have to guide the definition of core, depth, spacing of supports, the nature of the panel chords and the type of attachments to the substructure. There is a global need for interaction criteria, and presently empirical criteria will have to suffice.

Internal pressure often defines the core and core depth, the tension strength of the attachments and maximum internal "chord-wise" or "span-wise" loads in the face sheets. Symmetric face sheets are favored, if practical, as is symmetric panels in order to limit "secondary" deformations.

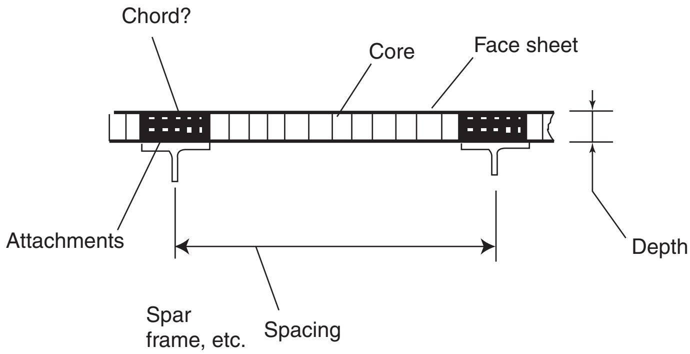

Figure 14.6. Honeycomb design variables.

An initial design that satisfies ultimate static strength requirements is a good starting point for the damage tolerance design phase for this concept also. It would also make a natural starting point for the following cycles of redesigns.

A starting point for the damage tolerance design has been established, and alternative approaches for special circumstances could always be considered, if practicality so demands. The considerations for examples of non-linear effects have been given their proper mentioning and it is now possible to identify the relation between internal and external loads, when the external loads are increased by a safety factor of 1.5. We will now, in this chapter focus on the composite skin–stringer concept for the rest of this chapter.

14.4.3. Skin–stringer; primary detail design

In order to take the next step, damage tolerance design, in the design of a specific PSE, we have to investigate the lay-ups of skin and stringer to produce a controlled stiffness and material distribution. Example 14.3 addresses one of the criteria for relative distribution of skin and stringer areas. This example focuses on one type of consideration and alerts the insightful reader to the fact that this is a case-to-case consideration and a general approach needs to be applied to what the design criteria should contain. Example 14.3 is based on a need not to have "unbuckled" skins below limit load levels.

Example 14.3: This example focuses on a specific end-load requirement, N^{tot}, and a maximum compression strain level of 0.0045. The criterion of unbuckled at limit can be expressed as,

$$0.003 \leq K\left(\frac{t}{w}\right)^2 \Rightarrow t > \mathrm{w}\sqrt{\frac{0.003}{K}}$$

where K is the buckling factor.

We now focus on determining a value of t for $w = 6 \approx K$, and we have, $t > 0.15$. We now select $t = 0.15$ and explore the load requirement $N^{\text{tot}} = 20$ k/in, and if we select a skin modulus of elasticity of 6 MSI, we find that the requirement for stringer modulus and total area is,

$\bar{t}$	E_{str} (MSI)
0.30	23.6
0.33	19.7
0.36	16.9
0.39	14.8

So the lightest answer is the highest stringer modulus possible, but driving the modulus in the other direction is the fail-safe condition of a failed stringer. We now assume that the rupture of the stringer only affects the strain field in the local skin between the two intact stringers adjacent to the failure. The average strain in the skin at the failure due to limit load is,

$$\frac{\varepsilon}{1.5}\left[1+\frac{AE_{\text{str}}}{2wtE_{\text{sk}}}\right]\leq \varepsilon_{\text{UA}}$$

which for the case

$$\varepsilon=\varepsilon_{\text{UA}} \quad \text{becomes} \quad \frac{A}{wt}\cdot\frac{E_{\text{str}}}{E_{\text{sk}}}=1\Rightarrow\frac{E_{\text{str}}}{E_{\text{sk}}}=\frac{t}{(A/w)}$$

We now assume that the previous requirement results in (for skin thickness) $t > 0.15$ which would yield the following results, if we assume that the end-load capability is maintained,

$$E_{\text{sk}}t\varepsilon\left[2.67+1.33\frac{A}{wt}\frac{E_{\text{str}}}{E_{\text{sk}}}\right]=\frac{60}{1.5}\Rightarrow t=\frac{2.21\cdot 10^3}{E_{\text{sk}}}$$

and some presently realistic modulus ratios result in,

E_{sk}	t	$\bar{t}$ *for* $k=3$	$k=2$	$k=1$
6	0.37	0.49	0.55	–
8	0.28	0.37	0.42	–
10	0.22	–	0.33	–
12	0.18	–	–	0.36

What this table shows is that one needs to have a good failure strain prediction for the cases under consideration, but the trends clearly indicate the difficulty here.

It leads one to consider the value of the two common criteria of unbuckled skins at limit and limit capability for a failed stringer. The alternative to fail-safety is the use of *A*-values for the design values, which does not appear to be a constructive choice without a re-evaluation of the design criteria and regulations for design values and rules that have carried over from the "metal era" and are incorporated in present composite practice.

The purpose of this example is to illustrate the difficulties in living with common practices from the metal world, and the complications arising because many of the regulations for structural design have not been updated for composites.

14.5. CRITICAL DAMAGE TOLERANCE DESIGN

With a starting point, for the PSE in question, of a chosen structural concept and a design with ultimate strength using an initial detectability criterion, it is time to introduce the total threat definition, so damage size requirements can be established. For illustration purposes, we will pick a PSE that has a linear relation between internal and external loads. This PSE is also accessible to "walk-around" inspections. Figure 14.7 describes the damage types to be considered.

For these types of damage, inspection methods must be identified so that damage size can be determined for the probability of non-detection of 10^{-3}. Then the next step is to assure a damage resistance that keeps initial damage to less or equal to region 4 damage size.

The values for the mean of residual strength associated with the different types of damage can be described as in Figure 14.8.

The general case of loading has bi-axial loads, shear and pressure. An empirical failure criterion can be created and used to reduce to a parametric representation. One form of these kinds of criteria is,

$$R = R(R_x, R_y, R_{xy}, P) = R_x + R_y^n + R_{xy}^m + P^k \tag{14.1}$$

Here the following definitions apply,

$$R_x = \frac{N_{x\mathrm{appl}}}{N_{x\mathrm{cr}}}$$

$$R_y = \frac{N_{y\mathrm{appl}}}{N_{y\mathrm{cr}}}$$

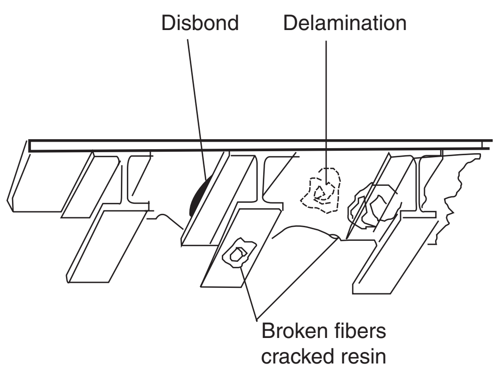

Figure 14.7. Damage types for skin–stringers.

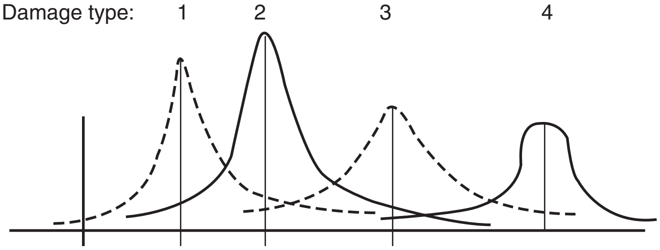

Figure 14.8. Residual strength distributions.

$$R_{xy} = \frac{N_{xy\text{appl}}}{N_{xy\text{cr}}}$$

$$P = \frac{p_{\text{appl}}}{p_{\text{cr}}}$$

A value of $R = 1.0$ implies failure, and the uni-axial equivalent case is,

$$R_x = 1.0 - R_y^n - R_{xy}^m - P^k \tag{14.2}$$

which produces an allowable value of $R_x \cdot N_{x\text{cr}}$, and,

$$\bar{t} = \frac{\text{LLR}}{R_x \cdot F_{x\text{cr}}}$$

The next step in this pursuit is to determine the size intervals starting with the location of "MUD." Example 14.4 illustrates the concerns.

Example 14.4: This study assumes normally distributed residual strength variables. Figure 14.9 describes the range.

The location of "MUD" determines the ultimate strength value, but it also sets the safety level for limit integrity. We start with region 2. It is a region of fast changing probabilities. The probability of detection can be written as,

$$P\left(\overline{H}D_2\overline{X}\right) = \sum_{i=1}^{n} P\left(\overline{H}\,\overline{X}_i D_{2i}\right) = \sum_{i=1}^{n} P\left(\overline{H}|D_{2i}\overline{X}_i\right) P\left(D_{2i}|\overline{X}_i\right) P\left(\overline{X}_i\right)$$

Here D_{2i} are n sub-intervals of region 2. A study of orders of magnitude for the detection, with MUD picked for a detection probability of 0.5, yields,

$$P\left(\overline{H}D_2\overline{X}\right) = 10^{-2} \cdot (0.9 \cdot 0.1 + 0.8 \cdot 0.2 + 0.7 \cdot 0.3 + 0.6 \cdot 0.4 + 0.5 \cdot 0.5) \approx 0.8 \cdot 10^{-2}$$

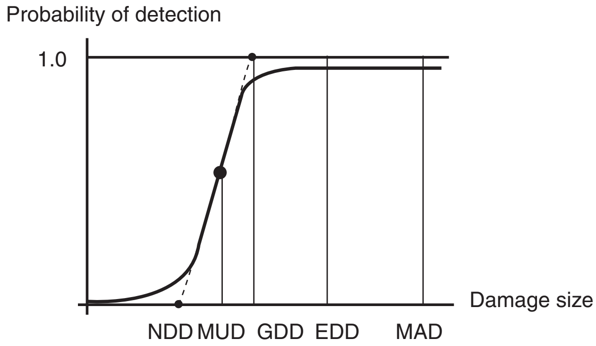

Figure 14.9. Damage size regions.

The probability of undetected loss of "Ultimate integrity" is (if *B*-values were used),

$$P(\overline{S}_{\mathrm{U}}) = 0.8 \cdot 10^{-3}$$

If we now assume that coefficient of variation (as damage is present) $C_{\mathrm{v}} = 0.10$, then we have for the undetected loss of "Limit integrity,"

$$P(\overline{U}_{\mathrm{L}}) = 0.8 \cdot 10^{-7}$$

This value would be inadequate for the safety levels we have been aiming at in the previous examples. We will return to the resolution of the "MUD" question after we have set the safety level required for this example.

14.5.1. Safety objective for damage tolerance critical structure

The vehicle safety objective, again should not be set less stringent than,

"One unsafe flight in 100 000 flights"

Assuming that innovation in structures will cause some surprises, we will assume that structure's share in mishaps is,

10 per cent

Further assuming that the share belonging to structural design can be assigned by Eq. (13.1) and equal shares are assumed, then the design share is,

$$\frac{1}{5}$$

Finally, we assume 50 PSEs, which gives each PSE the share,

$$\frac{1}{50}$$

So, for each PSE, the probability of an "unsafe state" is,

$$P(\overline{S}) = 10^{-5} \cdot 10^{-1} \cdot 2 \cdot 10^{-1} \cdot 2 \cdot 10^{-2} = 4 \cdot 10^{-9} \tag{14.3}$$

The first value of the level of safety associated with the ultimate was $0.8 \cdot 10^{-7}$ which requires a factor of,

$$\frac{0.8 \cdot 10^{-7}}{0.4 \cdot 10^{-8}} = 20$$

So, including region 3 in the ultimate definition would require the following change in the orders of magnitude as

$$P(\overline{S}_{L2}) = 0.8 \cdot 10^{-9}$$

A reasonable assessment of region 3 would lead to,

$$P(\overline{S}_{L3}) = 10^{-2} \cdot P(\overline{B}_{L}) \cdot (0.10 \cdot 0.09 + 0.09 \cdot 0.08 + 0.08 \cdot 0.07 + 0.07 \cdot 0.06 + 0.06 \cdot 0.05)$$

which for *B*-value ultimate allowables yields,

$$P(\overline{S}_{L3}) = 1.6 \cdot 10^{-9}$$

So for the damage range for ultimate allowable values extending to "GDD," the suggested values would be adequate to meet the safety level requirements, demonstrated in this section (14.5.1).

This would provide a rational approach to satisfy both ultimate and limit requirements.

14.5.2. Damage resistance and region 4

One way to serve the interests of safety, especially when damage growth rates can only be kept under specific finite rate, is to have a region 4. This region would contain the maximum damage sizes initially inflicted by all the identified threats. It would provide some grace before damage grows to a size for which the residual strength will violate damage tolerance integrity.

The upper limit for region 4 is "EDD." The determination of EDD has to be test based. The different threats have to be defined in terms of size and mass of impacting object, impact radius and velocity.

The resulting compromise between design and probability of inflicting maximum damage must be part of the Design Criteria and a definition of probability could be,

$$\Pr(\text{GDD} < D_{\text{s}} < \text{EDD}) < 10^{-3}$$

which would be a reasonable value considering that it involves rare circumstances. So, design criteria, for this example, would require that Damage Resistance Design would make Initial Accidental Damage size to less than "EDD."

14.5.3. Damage growth rate from region 4

The previous chapters contain investigations of maximum growth rates and have shown that a growth totally producing damage sizes in region 5 in three inspection intervals, after having started in region 4 produces a situation that is quite manageable from a safety standpoint. So, if we aim for a control period of three inspection intervals we find that,

The probability that a region 4 damage size grows into region 5 in one inspection interval is, $p = 0.24$;
The probability that a region 4 damage size grows into region 5 in two inspection intervals is, $p = 0.39$;
The probability that a region 4 damage size grows into region 5 in three inspection intervals is, $p = 0.5$.

The assumption is a uniform distribution between "no-growth" and maximum growth, in which case the probabilities from region 3 into region 4 are the same.

We now assume that the Damage Resistance design has been successful, so that we have a damage in region 4 at τ, as a worst case. We are at a location when accidental damage in service will be detected in "walk-around" inspections with a high probability.

14.5.4. Unsafe state as design constraint

We will now look into the consequences of the achieved damage resistance and maximum damage growth rate. We also will return to the expanded view of an unsafe state,

$$P(\overline{S}_T) = P(S_{\text{D}\tau}\overline{H}_\tau S_{\text{D}T}\overline{H}_T\overline{U}_T)$$

Here the state, $S_{D\tau}$, covers only region 4, while S_{DT} covers both region 4 and region 5. One expansion can be:

$$P(\overline{S}_T) = P(\overline{H}_\tau | S_{D\tau}) \cdot P(S_{D\tau}) \cdot P(\overline{H}_T | S_{DT}) \cdot P(\overline{B}_T | S_{DT}) \cdot P(S_{DT} | S_{D\tau}\overline{H}_\tau)$$

The total expansion is,

$$P(\overline{S}_T) = P(\overline{H}_\tau | S_{D\tau}) \cdot P(S_{D\tau}) \cdot \left[P(\overline{H}_T | S_{D4T}) \cdot P(\overline{B}_T | S_{D4T}) \cdot P(S_{D4T} | S_{D\tau}\overline{H}_\tau)\right.$$
$$\left. + P(\overline{H}_T | S_{D5T}) \cdot P(\overline{B}_T | S_{D5T}) \cdot P(S_{D5T} | S_{D\tau}\overline{H}_\tau)\right]$$

This can have the following order of magnitude for the desired, and based on previous assumptions in this chapter, we have,

$$P(\overline{S}_T) = 10^{-2} \cdot 10^{-2}[10^{-2} \cdot 10^{-3} \cdot 0.76 + 10^{-3} \cdot 10^{-1} \cdot 0.24] = 3.2 \cdot 10^{-9}$$

And the requirement in Eq. (14.3) is,

$$4 \cdot 10^{-9}$$

and the sizing is successful. If that had not been the case, it seems that the criteria damage would have been the first to change. So, for example one could have used a size that puts the allowable below,

$$0.87 \cdot \mu$$

Here μ is the mean value of the residual strength of the most critical damage type. A direct decrease of the allowable would reduce the probability of an unsafe state, which also could have been done by a direct increase in $\bar{t}$. This activity would then be repeated for all design points in the PSE.

14.6. TYPES OF DATA FOR DESIGN

The starting point, ultimate static strength, can be produced when the data shown in Figure 14.10 are available, together with 3D interaction law with parametric pressure. Tension, compression and shear allowable values are needed. For the damage tolerance design, a somewhat different format (see Figure 14.11) can be visualized.

It should be noted that Figure 14.11 only applies to one damage type and for one type of end-load. For this case, only *B*-values are being considered. If the data

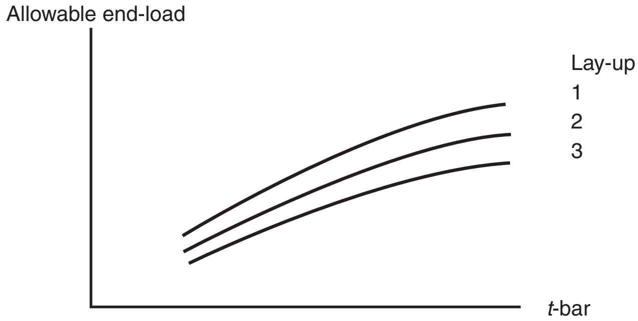

Figure 14.10. Ultimate allowable end-load for a specific concept.

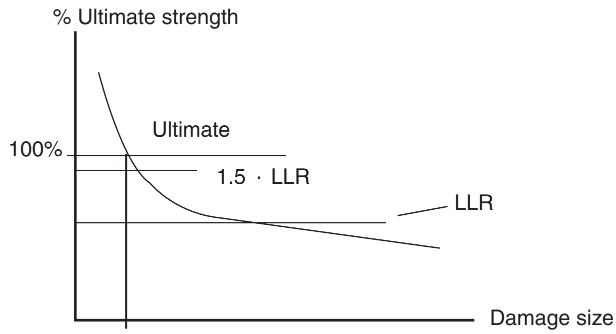

Figure 14.11. Per cent residual strength for a specific concept, lay-up and damage type.

are to be used for determining the critical damage type, means are often adequate, but the comparison offered by Figure 14.12 can often be helpful, and as different lay-ups can be most tolerant for different damage types, it may be efficient to only compare means while keeping the ultimate baselines in mind. A significant amount of focus is required in selecting realistic damage regions. Example 14.5 illustrates how the orders of magnitude are influenced by damage tolerance.

Example 14.5: We are assuming normal distributions and the effect of damage size is applied to the mean. The normal distribution can be written as,

$$\Phi(t) = \Phi\left(\frac{r - \mu\sqrt{(c/d_{\rm s})}}{\sigma}\right)$$

where $C = 2L$ and,

$$\mathrm{GDD} = 3L, \quad \mathrm{EDD} = 4L, \quad \mathrm{MAD} = 5L$$

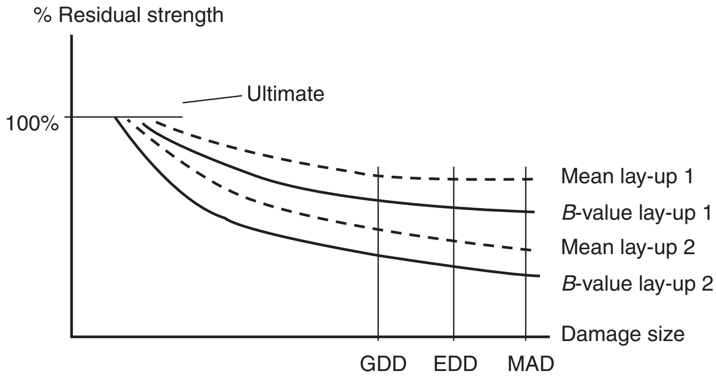

Figure 14.12. Residual strength for different lay-ups.

We now will compare B-values (for $C_v = 0.10$), for the square root to a cube root relation,

d_s	$\sqrt{\ }$	B	$\sqrt[3]{\ }$	B
$3L$	$1.22 \rightarrow 0.82\mu$	0.71μ	$1.14 \rightarrow 0.87\mu$	0.76μ
$4L$	$1.41 \rightarrow 0.70\mu$	0.61μ	$1.26 \rightarrow 0.79\mu$	0.69μ
$5L$	$1.58 \rightarrow 0.63\mu$	0.55μ	$1.36 \rightarrow 0.74\mu$	0.64μ

Now, if we assume LLR is set as the B-value at "MAD," then we can say that the probability of being less than LLR at $D_s = 4L$ is,

$$\Phi\left(\frac{0.5 - \sqrt{0.5}}{0.1}\right) = 0.018$$

and at

$$d_s = 3L \Rightarrow \Phi\left(\frac{0.5 - \sqrt{0.67}}{0.1}\right) = 0.0007$$

So region 4 is not in the running, in this case, for the probability of an unsafe state.

The size of region 5 could clearly be a baseline for B-value calculations, as it only loses 0.05μ in the interval for the moderate growth case and not much more for the "fast growth" case. What this example illustrates is that safety can be maintained at prescribed level even though the allowable values drop, and that it could be useful to compare actual residual strength reduction to square root and cube root effects in the selection of regions.

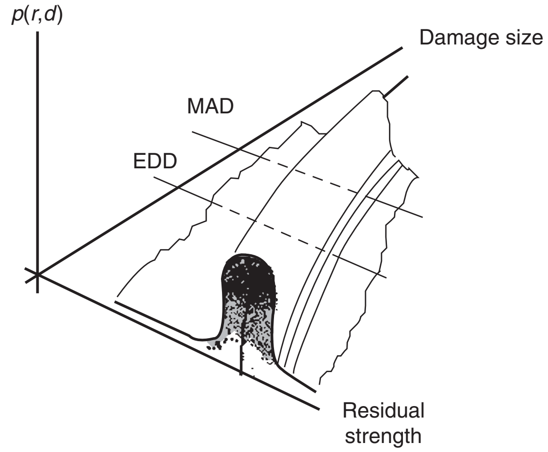

Figure 14.13. Joint probability density function for damage size and residual strength.

The development of design data depends on a realistic selection of damage size regions. The practice of using implied damage–flaw regions goes back to the definitions of preparations of coupons for ultimate strength and definition of cut-off strain for ultimate composites allowable values. In this application, however, it also should give reasonably stable detection probabilities, so a well-identified inspection program or program requirements must be part of the structural design for composites.

Figure 14.13, finally re-affirms the need to identify how external damage, internal damage, detection and residual strength interact to influence both damage tolerance and inspection with regard to method and frequency.

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Chapter 15
Design of Composite Structure

Polymeric composites constitute a large family of different materials. The family includes ‘‘Fiberglass,’’ ‘‘Kevlar,’’ ‘‘Graphite–Epoxies,’’ ‘‘Thermoplastics’’ (both high and moderate temperature variants), an endless variation of toughening agents, fiber sizing formulations, fibers, processing techniques and hybrids. From a design standpoint, it is a very heterogeneous family.

The design of composite structure and the detail design of composite structural concepts is a trip into innovation every time a ‘‘new’’ material species, a ‘‘new’’ process or a ‘‘new’’ structural concept is used. A custom-made design approach needs to be developed or adapted from a previous ‘‘choice,’’ because ‘‘new’’ often comes with different challenges. The development programs for ‘‘new’’ composite structures should come with a parallel design process development, including a special New Building Block Approach, NBBA, that embraces ‘‘scale-up’’ of design data. This type of ‘‘ad hoc’’ engineering is the price for a steady stream of improvements and possibly the approach for a future of ‘‘unlimited’’ opportunities.

A study of the defense sector reveals a consistent change of materials for new ‘‘vehicles.’’ ‘‘The best material always is the one we have not ‘screened’ yet.’’ The trend has carried over to the civilian sector with a steady search for and use of new materials. The history of composites teaches us that innovation will be the norm for a long time to come. The presence of applicable service experience will be the exception and uncertainty is a concept we will have to learn to contend with in risk management and uncertainty reduction through feedback and control processes. The activity must be part of the ‘‘Sustaining’’ of modern composites airframes and a commitment during the design phase.

Test programs must be supplemented with analytical scale-up of design data. The structural safety responsibility must be explicitly shared between regulations development, design, manufacturing, maintenance and operation. The feedback from monitoring of service data must be shared and distributed to source to make continuous safety improvements a reality. The fruits of feedback, if systematic, can be the source of an ever more proficient engineering community.

Finally, the design of composite structure must be conducted to explicit safety constraints applied to vehicle requirements kept continuously current with service experience, and updated as part of innovation.

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Appendix

A. A MODEL OF ULTIMATE INTEGRITY

A design point for the sizing of a composite PSE is defined by a design model, a set of damage types and a policy for "bolted repair." It is assumed that ultimate integrity is a requirement even when damage tolerance is critical. We will study a case of composite skin–stringers in Example A.1, and we will use the basis for design shown in the "Design Model" in Figure A.1. Composites design situations, especially in compression, are much more complicated than what is the case in the "metal world."

The situation in Example A.1 involves three types of "weaknesses" (design focus). They are:

Permanent fasteners;
Fasteners of "bolted repairs;"
Accidental damage locations and types.

All these "weaknesses" can be present in the model in Figure A.1 (repairs are assumed present). Figure A.1 has five damage types, 1–5. They are:

1. Debond between stringer and skin;
2. Delamination in skin;
3. Fiber and matrix breakage in skin and stringer flange;
4. Fiber and matrix breakage in skin and shear-tie;
5. Free stringer flange damage.

And fasteners are shown. All details are assumed to apply to location X. Figure A.1 includes events relating to fasteners, damage and repairs.

The probability of "acceptable ultimate integrity," AUI at location, X, U_X can be written as,

$$P(U_X) = P(U_F) \cdot P(U_R) \cdot P(U_{AD}) \Rightarrow P(\overline{U}_X) \approx P(\overline{U}_F) + P(\overline{U}_R) + P(\overline{U}_{AD}) \quad \text{(A.1)}$$

Here we have (temporarily omitting the buckling integrity):

U_F: AUI for all permanent fasteners;
U_R: AUI for "bolted repairs";
U_{AD}: AUI for all the accidental damage location involved.

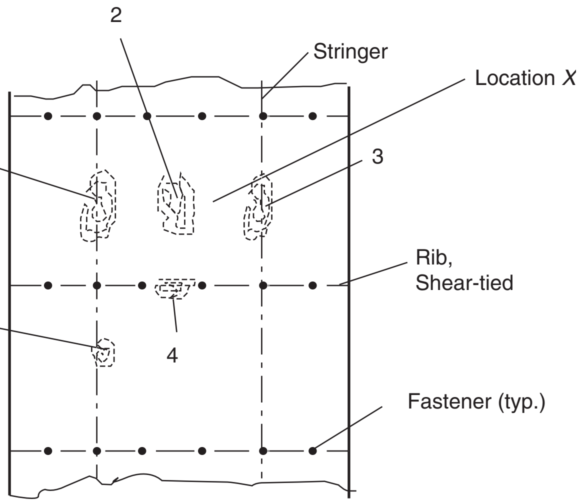

Figure A.1. Design model for location X, with fasteners and damage.

Example A.1 is focused on Eq. (A.1) for the special case of skin–stringers in compression.

Example A.1: The purpose of this example is to illustrate how a design model and damage scenario definitions make it possible to find the potentially dominating influences, especially when rare events are part of the design criteria.

We assume that no environmental degradation is present and that the equilibrium moisture level has been reached. The probabilities associated with the events relating to the "designed-in" fasteners are,

The probability of lost ultimate strength for all permanent fasteners becomes,

$$
\begin{aligned}
m \cdot \big[&P\big(\overline{B}_{U1}|H_{\mathrm{O}}T_{\mathrm{M}}M_{\mathrm{S}}\big)\cdot P(T_{\mathrm{M}}|H_{\mathrm{O}}M_{\mathrm{S}})\cdot P(H_{\mathrm{O}}|M_{\mathrm{S}})\\
&+P\big(\overline{B}_{U2}|\overline{H}_{\mathrm{O}}T_{\mathrm{M}}M_{\mathrm{S}}\big)\cdot P\big(T_{\mathrm{M}}|\overline{H}_{\mathrm{O}}M_{\mathrm{S}}\big)\cdot P(H_{\mathrm{O}}|M_{\mathrm{S}})\\
&+P\big(\overline{B}_{U3}|H_{\mathrm{O}}\overline{T}_{\mathrm{M}}M_{\mathrm{S}}\big)\cdot P\big(\overline{T}_{\mathrm{M}}|H_{\mathrm{O}}M_{\mathrm{S}}\big)\cdot P(H_{\mathrm{O}}|M_{\mathrm{S}})\\
&+P\big(\overline{B}_{U4}|\overline{H}_{\mathrm{O}}\overline{T}_{\mathrm{M}}M_{\mathrm{S}}\big)\cdot P\big(\overline{T}_{\mathrm{M}}|\overline{H}_{\mathrm{O}}M_{\mathrm{S}}\big)\cdot P\big(\overline{H}_{\mathrm{O}}|M_{\mathrm{S}}\big)\big] = P\big(\overline{U}_F\big) \qquad \text{(A.2)}
\end{aligned}
$$

The following events are involved:

H_{O}: Open-hole behavior;
T_{M}: Maximum temperature;
B_{Ui}: RS > ULR;

M_S: Equilibrium moisture content;
m: Number of fasteners;
a bar over a letter, signifying an event, indicates the complement.

The first term represents a "Hot-wet open-hole event probability," $P(U_{F1})$;
The second term, a "Hot-wet non-open-hole event probability," $P(U_{F2})$;
The third term, a "Non-hot, wet, open-hole event probability," $P(U_{F3})$;
The fourth term, a "Non-hot, wet, non-open-hole event probability," $P(U_{F4})$.

The following probabilities deal with events involved with "bolted repairs." Open-hole behavior is assumed and moisture equilibrium level has been reached. The probability of lost ultimate integrity of one "bolted repair" is,

$$P(\overline{U}_R) = n_R[P(\overline{U}_{R1}) + P(\overline{U}_{R2})] \quad \text{(A.3)}$$

Here n_R is the number of fasteners in one repair, and multiple repairs increase the number, no matter how it is done;

U_{R1} is a "hot-wet open-hole" event;
U_{R2} is a "non-hot-wet open-hole" event.

We assume n damage locations in the model, and each location can have t_i types. The probability of one type, j, of damage at one location, i, is:

$$P(\overline{U}_{ADij}) = P(\overline{B}_{ij}\overline{X}_i T_j D_{Uj}) = P(\overline{B}_{ij}|D_{Uj}T_j\overline{X}_i) \cdot P(D_{Uj}|T_j\overline{X}_i) \cdot P(T_j|\overline{X}_i) \cdot P(\overline{X}_i) \quad \text{(A.4)}$$

The following is the distribution of types:

	Type				
Location	1	2	3	4	5
1	•	•	•		
2		•	•		
3	•	•	•		
4		•		•	
5					•

The total contribution from "Accidental Damage" is:

$$P(\overline{U}_{AD}) = \sum_{i=1}^{5} \sum_{i}^{n_j} P(\overline{U}_{ADij}) \quad \text{(A.5)}$$

If all the terms are equal, we have,

$$P(\overline{U}_{\mathrm{AD}}) = 11 \cdot \overline{u}_{\mathrm{AD}}$$

here $\overline{u}_{\mathrm{AD}}$ represents one of the entries in the table.

Now, suppose that the tests of the seven design data types shown in the probability expressions; four for the permanent fasteners, two for the bolted repairs and one for accidental damage, have produced the following results:

$\mu_{\mathrm{F1}} = \mu$;
$\mu_{\mathrm{F2}} = 1.10\mu$;
$\mu_{\mathrm{F3}} = 1.20\mu$;
$\mu_{\mathrm{F4}} = 1.30\mu$;
$\mu_{\mathrm{R1}} = \mu$;
$\mu_{\mathrm{R2}} = 1.10\mu$;
$\mu_{\mathrm{AD}} = 1.10\mu$.

We assume that, all have normal distributions and that the standard deviations obey the assumptions. We assume that the structure was sized with μ (meaning that it is the internal design load for this location), then the table (below) provides the first factor in all the terms describing the probability of loss of ultimate integrity:

C_{v}	$1.10\mu \Rightarrow \Phi(-0.1/C_{\mathrm{v}})$	$1.20\mu \Rightarrow \Phi(-0.2/C_{\mathrm{v}})$	$1.30\mu \Rightarrow \Phi(-0.3/C_{\mathrm{v}})$
0.05	0.02	0.0003	10^{-9}
0.07	0.07	0.002	10^{-5}
0.10	0.16	0.022	0.001

We now assume that the "fastener-related" allowable values have $C_{\mathrm{v}} = 0.07$ and the accidental damage data have $C_{\mathrm{v}} = 0.10$.

A realistic first assessment of the probability of loss of structural integrity at a specific location could look like the following,

$$P(\overline{U}_X) = m(\overline{p}_1 \cdot 10^{-2} \cdot 10^{-1} + \overline{p}_2 \cdot 10^{-2} \cdot 0.9 + \overline{p}_3 \cdot 1 \cdot 10^{-1} + \overline{p}_4 \cdot 1 \cdot 0.9)$$
$$+ n_{\mathrm{R}}(\overline{p}_5 \cdot 10^{-2} \cdot 10^{-1} \cdot 10^{-2} + \overline{p}_6 \cdot 1 \cdot 10^{-1} \cdot 10^{-2}) + 11 \cdot \overline{p}_7 \cdot 0.9 \cdot 0.2 \cdot 10^{-2}$$

Here the first term is an assessment of Eq. (A.2), the second Eq. (A.3) and the third Eq. (A.5). An evaluation of the postulated test results yields,

$$P(\overline{U}_X) = m(0.5 \cdot 10^{-3} + 0.063 \cdot 10^{-2} + 0.002 \cdot 10^{-1} + 0.9 \cdot 10^{-5})$$
$$+ n_{\mathrm{R}}(0.5 \cdot 10^{-5} + 0.07 \cdot 10^{-3}) + (11 \cdot 0.16 \cdot 0.5 \cdot 10^{-2})$$

which yields,

$$P(\overline{U}_X) = 1.34 \cdot 10^{-2} + 0.08 \cdot 10^{-2} + 0.88 \cdot 10^{-2} = 2.3 \cdot 10^{-2}$$

The values used for m and n is 10. It should however be noted that for a "perfect," permanent fastener installation process m would tend to one, ($m \to 1$). In which case $P(\overline{U}_X)$ would become about a half of the above.

Here the first term in the answer represents "permanent fasteners," the second "bolted repair" and the third "accidental damage."

It is noteworthy that the allowable values, for "Hot-wet open-hole compression" and "Accidental Damage," are based on the mean values of strength and used in a balanced design, meaning that they influence all three terms of the answer. It would appear that a situation like the one described earlier, if backed up with a realistic "Damage Tolerance Design (limit integrity)," would represent a respectable safety level.

So, e.g. if the level of structural integrity was normally distributed, then we could write,

$$\Phi\left(\frac{u-\mu}{\sigma}\right) = 2.3 \cdot 10^{-2} \Rightarrow \frac{(u/\mu)-1}{C_{\mathrm{v}}} = -2.00 \Rightarrow \frac{u}{\mu} = 1 - 2C_{\mathrm{v}}$$

as damage is involved, we assume $C_{\mathrm{v}} = 0.10$ yielding,

$$\Phi\left(\frac{(0.8/1.5)-1}{0.1}\right) = \Phi(-4.67) = 1.5 \cdot 10^{-6}$$

which would represent the probability of loss of equivalent damage tolerance level of safety.

If we now assume that the classical buckling evaluation results in a mean, that is 1.1μ, then we find that the contribution to the probability of loss of ultimate integrity is 0.07, which would yield a total value of 0.093, which is slightly better than a B-value. If we consider the fact that in the traditional "metal design world" the critical design driver is stability, we find that a structure is considered safe, if the probability of loss of ultimate integrity is less or equal to 0.1 (B-value). From that we can conclude that the situation described for composites is "Safe," if a mean value were used in sizing of the design.

From this example, it appears that a case-by-case assessment of the actual practical design situation and the identification of the model to be used at every location is a must, in order to achieve a proper safety level in the design.

As also was demonstrated, there are cases when the mean of the strength is quite an adequate basis for allowable values, especially when a detailed damage tolerance design is done. So, composites require an integral interaction between criteria and "local" design realities, on a case-by-case basis, which fits in well with the concept of PSE specific design criteria.

B. A COMPARISON BETWEEN METAL AND COMPOSITE PANELS

In the metal world, we produce *B*-values for panels by testing of panels with loads, environments and conditions that are well-known and "deterministic in nature." In the composite world, the practice of panel testing to produce allowables has not "caught on."

There is a fundamental difference that must be accounted for. The example in Appendix A has a number of effects that are not considered in the metal world. So, e.g. for the "designed-in" fasteners, there is a random behavior that occasionally makes the failure appear to have the characteristics of an "open-hole." The critical load case is assumed to be compatible with the maximum structural temperature, even though often a significant cool down has taken place, the value depending on time from take-off. Structure critical for landing conditions seems severely penalized, if maximum temperature is assumed, and a whole set of intermediate conditions is bound to need evaluation from the standpoint of temperature. For locations where the probability of maximum temperature is large, it should be expected the equilibrium moisture content would be low, and the practice is characterized as "the use of very unlikely environments" for design.

Detail, deterministic analyses of all the combinations seems self-defeating, when a probabilistic assessment could be done. One might ask, what the compliance demonstration should look like? However, before we do try to address that challenge, we should address the other to random effects. Number one is the presence of bolted repairs, especially in region where "designed-in" fasteners are not present, is truly a random effect. Finally, the presence of damage, type and location has a large influence on the ultimate integrity, but is random.

So while, in the metal world the probability of loss of ultimate integrity is totally tied to the panel allowable without damage and representing well-defined typical situations (fatigue damage is not included), the composite world has created a totally different situation by including random behavior, random damage and repair. So, one would expect that compliance demonstrations would be very different, maybe a combination of validated random influences, with some test validation for isolated effects to validate the means used in Appendix A.

At any rate, panel testing with combinations of some of the random effects is of very little value in the production of design data. Some testing of panels is, however, very important for design data values. As a minimum, one would expect to see ‘‘as-designed’’ panels, repaired panels and panels with damage used in the testing for design data and the compliance demonstration and methods validation.

The safety parameter worth considering is the ‘‘Probability of loss of integrity,’’ which would provide consistent adaptation of ‘‘metal world’’ practices in maintaining levels of safety. It could be argued that the added detail requirements and random nature of these for composites would result in a ‘‘Probability of Safe Structure’’ that would be more complicated than the one defined in Chapter 1. One version would be,

$$P(DMIOR) = P(D|MIOR)P(R_2|R_1MIO)P(R_1|MIO)P(M|IO)P(I|O)P(O) \tag{B.1}$$

where

D: Safe design;
R: Safe requirements, $R = R_1 \cdot R_2$;
R_2: Safe predicted internal loads requirements;
R_1: Safe regulations;
M: Safe manufacturing;
I: Safe maintenance, including safe inspections;
O: Safe operation.

The first factor in Eq. (B.1) deals with safe design, given all the other events; the second deals with safe internal loads predictions under identified given conditions; the third with the probability of safe regulations, given safe manufacturing, safe maintenance and safe operation. Resulting in an even smaller share of the probability of safe structure available for design, which certainly would be consistent with a prudent approach to safe innovation. A realistic, practical approach to New designs of composite structure must recognize the existence of many Uncertainties and must proceed with an approach that emphasizes risk and safety.

The probability that present regulations are unsafe, when applied to composites, is relatively large and of grave concern, as one would expect that all the differences pointed out in the appendix would lead to new requirements.

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